Determining the Axial Radius of the Nucleon from Data on Pion Electroproduction

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We discuss a low-energy theorem of threshold pion electroproduction which allows one to determine the axial root-mean-square radius r_A of the nucleon. We show that at the same order where the radius appears, pion loops induce a small correction to the momentum dependence of the electric dipole amplitude $E_{0+}^{(-)}$. This correction amounts to a decrease of the axial mean-square radius by 5% from the electroproduction data and allows one to explain the small discrepancy to the values determined from (anti)neutrino-proton scattering data.

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The nucleon is an extended object, as revealed in elastic electron scattering by its electromagnetic root-mean-square radii. It also possesses an axial mean-square radius, r_A, related to the distribution of spin and isospin. The latter has been determined so far by two different methods. The first is (anti)neutrino-proton scattering [1-3] and the second is threshold pion electroproduction [4,5]. It is common to parametrize the axial form factor of the nucleon by a dipole form, $G_A(k^2)=(1-k^2/M_A^2)^{-2}$, with k^2 the four-momentum transfer squared. Neutrino scattering experiments lead to a cutoff mass $M_A=1.03-1.09$ GeV, i.e., $r_A=\sqrt{12}/M_A=0.63-0.66$ fm. On the other hand, electroproduction experiments systematically give somewhat higher values, the most accurate one being $M_A=1.15\pm0.10$ GeV ($r_A=0.59$ fm) [5,6]. This means that in electroproduction one seems to see a smaller axial radius. Generally, this discrepancy is not taken seriously since the central values overlap within the error bars (which are larger in the electroproduction case). In this Letter, we wish to point out that this discrepancy is a real, i.e., *physical* effect. Furthermore, it can be calculated in a model-independent manner.

Our starting point is the ancient low-energy theorem due to Nambu, Lurié, and Shrauner [7], which relates the S-wave multipole $E_{0+}^{(-)}$ of charged pion electroproduction at threshold to the normalized axial form factor of the nucleon, $G_A(k^2)$, for vanishing pion mass ($M_\pi=0$),

$$E_{0+}^{(-)}(M_\pi=0,k^2)=\left[1-\frac{k^2}{4m^2}\right]^{1/2}\frac{eg_A}{8\pi F_\pi}\left\{G_A(k^2)+\frac{k^2}{4m^2-2k^2}G_M^V(k^2)\right\}, \tag{1}$$

with $G_M^V(k^2)$ the isovector magnetic form factor of the nucleon, g_A the axial-vector coupling constant ($g_A=1.26$), $F_\pi=93$ MeV the pion decay constant, and $m=939$ MeV the nucleon mass. Expanding Eq. (1) to order k^2 one has

$$E_{0+}^{(-)}(M_\pi=0,k^2)=\frac{eg_A}{8\pi F_\pi}\left\{1+\frac{k^2}{6}r_A^2+\frac{k^2}{4m^2}\left[\kappa_V+\frac{1}{2}\right]+O(k^3)\right\}, \tag{2}$$

with $\kappa_V=3.71$ the isovector anomalous magnetic moment of the nucleon. Notice that to this order in the energy expansion, one only sees the leading term in the expansion of the isovector magnetic form factor. Clearly, from Eq. (2) one can deduce r_A if one measures $E_{0+}^{(-)}$ sufficiently close to threshold [8] (via, e.g., the reaction $\gamma^* p\to\pi^+ n$).

Our aim is to systematically calculate the corrections to the chiral limit ($M_\pi=0$) in a model-independent fashion. In particular, we are interested in all corrections up to and including $O(M_\pi^2)$. For that we make use of baryon chiral perturbation theory in the heavy-mass formulation [9-11]. Nucleons are treated as very heavy fields and it is therefore possible to eliminate the troublesome baryon mass term to leading order in the effective field theory. This allows for a consistent power counting scheme, which states that loop corrections are suppressed by powers of q^2, with q denoting a genuine small momentum or meson mass [12]. The effective theory to lowest order is nothing but the nonlinear σ model coupled to nucleons. Calculating tree diagrams, one recovers the well-known current algebra results. However, one has to include loops in the effective theory. First, tree diagrams are real and thus unitarity is violated. This can be cured in a perturbative fashion by including loops. Second, the pion cloud surrounding the nucleon becomes long ranged in the chiral limit. This leads to large nonanalytic corrections in certain observables. To renormalize the sometimes infinite loop contributions, one has to add local counterterms of higher order in the derivative expansion [13]. These terms are accompanied by *a priori* unknown coefficients which have to be determined phenomenologically. The explicit form of the Lagrangian of interacting pions, nucleons, and photons (as well as other external fields) is spelled out in detail in Ref. [10]. In what follows, we will work in the one-loop approximation which is of sufficient accuracy here, as will be discussed later on.

A straightforward calculation of all relevant one-loop diagrams contributing to $E_{0+}^{(-)}$ at threshold gives (to order q^2)

$$E_{0+}^{(-)}(M_\pi\neq 0,k^2)=\frac{eg_A}{8\pi F_\pi}\left\{1+\frac{k^2}{6}r_A^2+\frac{k^2}{4m^2}\left(\kappa_V+\frac{1}{2}\right)-\frac{M_\pi}{m}+CM_\pi^2 +\frac{M_\pi^2}{8\pi^2F_\pi^2}\int_0^1 dx\int_0^\pi dy\ln\left(1-x^2+y^2+\frac{k^2}{M_\pi^2}x(x-1)\right)\right\}+O(q^3)\,. \qquad (3)$$

Let us discuss the various terms appearing in Eq. (3). Naturally, we recover the ones already present in the chiral limit, Eq. (2). It is important to observe that the axial radius is finite for vanishing pion mass. The first new term is a purely kinematical (recoil) correction. The term CM_π^2 subsumes various corrections whose explicit forms are not needed in detail for the following discussion. These are the chiral logarithms $\sim \ln M_\pi^2$ which come from the loops, $1/m^2$ suppressed (kinematical or recoil) corrections, and a set of higher-order contact terms necessary to renormalize the loops. However, all these contributions do not depend on the momentum transfer k^2. The last term in (3), the integral, is most interesting. It comes entirely from the so-called triangle and tadpole (with the three pions coupling to the nucleon at one point) diagrams (and their crossed partners), which play a prominent role in pion photoproduction and electroproduction [14,15]. In the physical region $k^2\leq 0$ this contribution vanishes identically at $M_\pi=0$ and so do the other new terms. Thus one recovers the Nambu-Lurié-Shrauner result for the chiral limit. Matters are different at nonvanishing pion mass. Let us work out the coefficient of the term proportional to k^2 since it contains the information about the axial radius. In that case, the integral appearing in (3) can be done analytically. The coefficient of the k^2 term reads as

$$\frac{1}{6}r_A^2+\frac{1}{4m^2}\left(\kappa_V+\frac{1}{2}\right)+\frac{1}{128F_\pi^2}\left(1-\frac{12}{\pi^2}\right). \qquad (4)$$

Notice that the last term in Eq. (4) is a one-loop effect which cannot be canceled by higher loop contributions. It therefore constitutes a model-independent new term at order k^2, not taken into account so far. Higher loop corrections are suppressed by powers of M_π/m or $M_\pi^2/16\pi^2F_\pi^2$ which are small. The formal reason for the appearance of this novel contribution is that one cannot interchange the order of taking the derivative at $k^2=0$ and the chiral limit $M_\pi\to 0$. Clearly, this term has consequences for the value of the axial radius extracted from the electroproduction data. Indeed, since κ_V is known, previous determinations have "measured" the modified axial radius

$$\tilde{r}_A^2=r_A^2+\frac{3}{64F_\pi^2}\left(1-\frac{12}{\pi^2}\right). \qquad (5)$$

The correction term appearing in Eq. (5) has a value of -0.0456 fm^2, which is a 10% correction to a typical mean-square radius of $r_A^2=0.45$ fm^2. To be specific, consider the mean result of a neutrino scattering experiment, $M_A=1.05$ GeV or $r_A^2=0.42$ fm^2. Because of the new term at order k^2, in electroproduction one effectively sees a smaller axial radius, $\tilde{r}_A^2=0.38$ fm^2, corresponding to $M_A=1.11$ GeV. This is at the heart of the discrepancy between the neutrino-scattering and electroproduction data discussed in the introduction. Stated differently, if one takes into account in the analysis of the pion electroproduction data the second term in Eq. (5), one finds a value for the axial radius of the nucleon completely consistent with the one from the neutrino data.

Finally, we wish to point out that new precise electroproduction experiments are planned and approved (e.g., at MIT-Bates) which will measure very close to the photon point [16]. These experiments should allow for a more precise determination of the axial radius of the nucleon and, obviously, in their analysis the pion loop corrections discussed here will have to be taken into account. A more detailed discussion of pion electroproduction in the framework of chiral perturbation theory (including also the higher-order terms in the momentum expansion) will be given in Ref. [17].

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