On the Strangeness –1 S-Wave Meson-Baryon Scattering

José A. Oller
Departamento de Física, Univ. Murcia


1. Introduction. Interest.
2. UCHPT
3. S-Wave, S=−1 Meson-Baryon Scattering
4. Scattering
5. Spectroscopy
6. Conclusions
1. INTRODUCTION. INTEREST.

$\overline{K}N$ SCATTERING, TEN TWO BODY COUPLED CHANNELS:

\[ \pi^0 \Lambda \pi^0 \Sigma^0 \pi^- \Sigma^+ \pi^+ \Sigma^- K^- p \overline{K}^0 p \eta \Lambda \pi^0 \Sigma^0 K^0 \Xi^0 K^- \Xi^+ \]

\[ 8 \times 8 = 1 + 8_s + 8_a + 10 + \overline{10} + 27 \]

The representations 1, $8_s$, $8_a$ and 27(exotic) give rise to resonances.

- Potential Models, Quark Models, (Chiral) Bag Models, etc
- CHPT+Unitarization (UCHPT)
  - Kaiser, Siegel, Weise NPA594,325(’95)
  - Oset, Ramos NPA635,99(’98)
  - Meissner, JAO PLB500,263(’01)
  - Lutz, Kolomeitsev NPA700,193(’02)
  - Borasoy, Nissler, Weise PRL94,213401 (05), EPJA25,79(’05), etc
Renewed interest with the precise measurement by DEAR Coll. of strong shift and width of kaonic hydrogen 1s energy level

G. Beer et al., PRL94,212302(’05)

Unstable Meissner,Raha,Rusetsky EPJ C35,349(’04); Borasoy,Nissler,Weise PRL94,213401(05), EPJA25,79(05) pointed out a possible inconsistency between DEAR and previous scattering data. SU(3) chiral dynamics results agree with KEK but disagrees with the factor 2 more precise DEAR measurement.
\[ E_{1s} = E_{1s}^{em} + \epsilon_{1s}, \epsilon_{1s} \text{ is complex} \]

**Deser Formula** \[ \epsilon_{1s} = -2\alpha^3\mu_C^2 T_{K^-p} \]

**Precise knowledge**

\[ \epsilon_{1s} \leftrightarrow T_{K^-p} \text{ at threshold} \]

Meissner, Raha, Rusetsky *EPJ C*35,349(’04) include isospin breaking correction on the Deser formula up to an including \( O(\alpha^4, \alpha^3(m_u-m_d)) \sim 9\% \)

Cusp Effect: \( \sim 50\% \ O(\delta^{1/2}) \)

Coulomb Effects: \( \sim 10 - 15 \% \)

Vacuum Polarization: \( \sim 1\% \)

\[ \delta \sim \alpha \sim m_u - m_d \]

DEAR/SIDDHARTA Coll. Aims to finally measure it up to eV level, a few percent (nowadays the precision is 20\%).

http://www.lnf.infn.it/esperimenti/dear/DEAR_RPR.pdf
Other interesting finding for which a precise knowledge of $\bar{K}N$ scattering is of foremost importance:

- Nature of $\Lambda(1405)$, problems in lattice QCD. Dynamically generated resonance.
- Two poles making up the $\Lambda(1405)$
  Meissner, J.A.O. PLB500, 263 (’01); Jido, Oset, Ramos, Meissner, J.A.O, NPA725(03)181
  Magas, Oset, Ramos PRL95, 052301 (’05); S. Prakhov et al. (Crystall Ball Coll.),
  PRC70, 034605(’04);
- Strangeness content of the proton and large pion-nucleon sigma terms,
  $\langle p|\bar{s}s|p \rangle$ strange proton-scalar form factor related by unitarity with $\bar{K}N$ amplitudes.
2. UNITARY CHPT (UCHPT).

1. A systematic scheme able to be applied when the interactions between the hadrons are not perturbative (even at low energies).
   - **Meson-meson processes**, both scattering and (photo)production, involving $I=0,1,1/2$ S-waves, $J^{PC}=0^{++}$ (vacuum quantum numbers) $I=0$ $\sigma(500)$ - really low energies
     Not low energies - $I=0$ $f_0(980)$, $I=1$ $a_0(980)$, $I=1/2$ $\kappa(700)$. Related by SU(3) symmetry.
   - Processes involving $S=-1$ (strangeness) S-waves meson-baryon interactions $J^P=1/2^-$. $I=0$ $\Lambda(1405)$, $\Lambda(1670)$, $I=1$ $\Sigma(1670)$, possible $\Sigma(1400)$, etc...
     One also finds other resonances in $S=-2$, 0, +1, and even with $I=2$...
   - Processes involving scattering or production of, particularly, the lowest Nucleon-Nucleon partial waves like the $^1S_0$, $^3S_1$ or P-waves. *Deuteron*, Nuclear matter, *Nuclei*.

2. Then one can study:
   - Strongly interacting coupled channels.
   - Large unitarity loops.
   - Resonances.
4. This allows as well to use the Chiral Lagrangians for higher energies. (BONUS)

5. Since one can also use the chiral Lagrangians for higher energies it is possible to establish a connection with perturbative QCD, $\alpha_s(4 \text{ GeV}^2)/\pi \approx 0.1$. (OPE). E.g. providing phenomenological spectral functions for QCD Sum Rules (going definitively beyond the sometimes insufficient hadronic scheme of narrow resonance+resonance dominance). Jamin, Pich, JAO

\[ V_{us} : \text{JHEP 0402, 047 ('04)} \]

\[ m_{u,d,s} : \text{EPJ C24, 237 ('02); hep-ph/0605095} \]

6. The same scheme can be applied to productions mechanisms. Some examples: !

- **Photoproduction:** $\gamma \gamma \rightarrow \pi^0\pi^0, \pi^+\pi^-, K^+K^-, K^0\bar{K}^0, \pi^0\eta; D \rightarrow 3\pi, K 2\pi,$ ...
  $\gamma p \rightarrow K^+ \Lambda(1405); (\gamma, \pi\pi); \gamma d \rightarrow d; \gamma NN \rightarrow NN; \gamma d \rightarrow \gamma d; ...$

- **Decays:** $\phi \rightarrow \gamma \pi^0\pi^0, \pi^0\eta, K^0\bar{K}^0; J/\Psi \rightarrow \phi(\omega) \pi\pi, KK; f_0(980) \rightarrow \gamma\gamma; \text{branching ratios} ...$

JAO PRD 71, 054030 ('05) on D $\rightarrow 3\pi, K 2\pi$ and $D_s \rightarrow 3\pi$, and references therein
Chiral Perturbation Theory
Weinberg, Physica A96,32 (79); Gasser, Leutwyler, Ann.Phys. (NY) 158,142 (84)

QCD Lagrangian

\[ u, d, s \text{ massless quarks} \]
\[ \text{SU}(3)_L \otimes \text{SU}(3)_R \]

Hilbert Space

Physical States

Spontaneous Chiral Symmetry Breaking

\[ \text{SU}(3)_V \]

Goldstone Theorem

Octet of massless pseudoscalars
\[ \pi, K, \eta \]

Energy gap
\[ \rho, K^*, \phi, K_0^*(1450) \]

Non-zero masses
\[ m_p^2 \propto m_q \]

\[ m_q \neq 0. \text{ Explicit breaking of Chiral Symmetry} \]

\[ L = L_2 + L_4 + \ldots \]
\[ \frac{L_4}{L_2} = O\left(\frac{p^2}{\Lambda_{\text{CHPT}}^2}\right) \]
\[ \Lambda_{\text{CHPT}} \approx 1 \text{ GeV} \approx M_\rho \]
\[ \approx 4\pi f_\pi \approx 1 \text{ GeV} \]
• Enhancement of the unitarity cut that makes definitively smaller the overall scale $\Lambda_{\text{CHPT}}$ in meson-baryon scattering with strangeness:

![Arbitrary Meson-Baryon Vertex]

- Presence of large masses compared with the typical low three-momenta (Baryon+Kaon masses) drive the appearance of the $\Lambda(1405)$ close to threshold in $\bar{K}N$ scattering.

  This also occurs similarly in Nucleon-Nucleon scattering with the nucleon mass
Let us keep track of the kaon mass, $M_K \approx 500$ MeV

We follow similar arguments to those of S. Weinberg in NPB363,3 (’91) respect to NN scattering (nucleon mass).

Unitarity Diagram

$$\int \frac{dq^0}{(k^0 - q^0 + i\epsilon)(q^0 + E(q) - i\epsilon)(q^0 - E(q) + i\epsilon)}$$

Unitarity enhancement for low three-momenta:

$$\frac{1}{k^0 - E(q)} \frac{1}{2E(q)} \approx \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}$$

Around one order of magnitude in the region of the $\Lambda(1405)$ region, $|q| \approx 100$ MeV
Let us keep track of the kaon mass, $M_K \approx 500$ MeV
We follow similar arguments to those of S. Weinberg in NPB363,3 (’91) respect to NN scattering (nucleon mass).

Let us take now the crossed diagram

$k \rightarrow -k$

$\frac{1}{k^0 + E(q)} \frac{1}{2E(q)} \approx \frac{1}{4M_K^2}$

Unitarity enhancement for low three-momenta:

$\frac{2M_K}{q}$

Unitarity Diagram

$\frac{1}{k^0 - E(q)} \frac{1}{2E(q)} \approx \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}$
In all these examples the unitarity cut (sum over the unitarity bubbles) is enhanced.

UCHPT makes an expansion of an ```Interacting Kernel´´ from the appropriate EFT and then the unitarity cut is fulfilled to all orders (non-perturbatively)
Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

$$\text{Im } T_{ij} = \sum_k T_{ik} \rho_k T^*_{kj} \quad \text{\rightarrow} \quad \text{Im } T_{ij}^{-1} = -\rho_i \delta_{ij}$$  

Unitarity Cut

We perform a dispersion relation for the inverse of the partial wave (the discontinuity when crossing the unitarity cut is known):

$$T_{ij}^{-1} = R_{ij}^{-1} + \delta_{ij} \left( g(s_0)_i - \frac{s - s_0}{\pi} \int_{s_{th,i}}^{\infty} \frac{\rho(s')_i}{(s' - s - i0^+)(s' - s_0)} ds' \right)$$

The rest

\[ g(s)_i : \text{ Single unitarity bubble] \]
\[ g(s) = \frac{1}{4\pi^2} \left( a_{SL} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right) \]

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = \left[ I + R \cdot g \right]^{-1} \cdot R \quad \sigma(s) = \frac{2q}{\sqrt{s}} \]

1. \( T \) obeys a CHPT/alike expansion
\[ g(s) = \frac{1}{4\pi^2} \left( a_{SL} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right) \]

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = \left[ I + R \cdot g \right]^{-1} \cdot R \]

1. \( T \) obeys a CHPT/alike expansion \( T = T_1 + T_2 + T_4 + \ldots \)

2. \( R \) is fixed by matching algebraically with the CHPT/alike expressions of \( T \), \( R = R_1 + R_2 + R_3 + \ldots \)

In doing that, one makes use of the CHPT/alike counting for \( g(s) \)

The counting of \( R(s) \) is consequence of the known ones of \( g(s) \) and \( T(s) \)
\[ g(s) = \frac{1}{4\pi^2} \left( a_{SL} + \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \right) \]

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = [I + R \cdot g]^{-1} \cdot R \quad \sigma(s) = \frac{2q}{\sqrt{s}} \]

1. \( T \) obeys a CHPT/alike expansion \( T = T_1 + T_2 + T_4 + \ldots \)

2. \( R \) is fixed by matching algebraically with the CHPT/alike expressions of \( T \), \( R = R_1 + R_2 + R_3 + \ldots \)

In doing that, one makes use of the CHPT/alike counting for \( g(s) \)

The counting of \( R(s) \) is consequence of the known ones of \( g(s) \) and \( T(s) \)

3. The CHPT/alike expansion is done to \( R(s) \). Crossed channel dynamics is included perturbatively.
Historically, the first approach to apply a Chiral expansion to an interacting \textbf{KERNEL} was:


The Chiral expansion was applied to the set of two nucleon irreducible diagrams, THE POTENTIAL, which was then iterated through a Lippmann-Schwinger equation.

\begin{center}
\begin{tikzpicture}
    \node[circle,fill=black] (1) at (0,0) {};
    \node[circle,fill=black] (2) at (1.5,0) {};
    \node[circle,fill=black] (3) at (3,0) {};
    \node[circle,fill=black] (4) at (4.5,0) {};
    \node[circle,fill=black] (5) at (6,0) {};
    \node[circle,fill=black] (6) at (7.5,0) {};
    \node[circle,fill=black] (7) at (9,0) {};
    \node[circle,fill=black] (8) at (10.5,0) {};
    \node[circle,fill=black] (9) at (12,0) {};
    \node[circle,fill=black] (10) at (13.5,0) {};
    \draw (1) -- (2);
    \draw (2) -- (3);
    \draw (3) -- (4);
    \draw (4) -- (5);
    \draw (5) -- (6);
    \draw (6) -- (7);
    \draw (7) -- (8);
    \draw (8) -- (9);
    \draw (9) -- (10);
    \draw (10) -- (11);
\end{tikzpicture}
\end{center}

The solution to the \textbf{LS equation} is \textbf{NUMERICAL}

Further regularization is needed when solving the LS equation (cut-off dependence) so that the new divergences are not reabsorbed by the counterterms introduced in $V$

3. S-WAVE, S=-1 MESON-BARYON SCATTERING

J. Prades, M. Verbeni, JAO PRL95, 172502(05), PRL96, 199202(06) (Reply)
J.A. Oller, EPJA 28, 63(2006)

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = \left[ I + R \cdot g(s) \right]^{-1} \cdot R(s) \]

\[ R = R_1 = T_1 \quad \text{LEADING ORDER, } O(p) \]

\[ R = R_1 + R_2 = T_1 + T_2 \quad \text{NLO, } O(p^2) \]

for \( O(p^3) \) and higher \( R_n \neq T_n \)

\[ O(p) \quad \text{Seagull} \]
\[ O(p^2) \quad \text{Direct} \]
\[ O(p^2) \quad \text{Crossed} \]
$\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ Chiral Lagrangians

$$
\mathcal{L}_1 = \langle i \bar{B} \gamma^\mu [D_\mu, B] \rangle - m_0 \langle \bar{B} B \rangle \\
+ \frac{D}{2} \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{F}{2} \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle,
$$

$D = 0.8, F = 0.46, m_0 =$ proton mass in SU(3) chiral limit

$$
\mathcal{L}_2 = b_0 \langle \bar{B} B \rangle \langle \chi_+ \rangle + b_D \langle \bar{B} \{\chi_+, B\} \rangle + b_F \langle \bar{B} [\chi_+, B] \rangle \\
+ b_1 \langle \bar{B} [u_\mu, [u^\mu, B]] \rangle + b_2 \langle \bar{B} \{u_\mu, \{u^\mu, B\} \} \rangle \\
+ b_3 \langle \bar{B} \{u_\mu, [u^\mu, B]\} \rangle + b_4 \langle \bar{B} B \rangle \langle u_\mu u^\mu \rangle + \cdots.
$$

$U = e^{i\phi/f}, U = u^2, v = e^{i\phi/2f}, u_\mu = iu^\dagger (\partial_\mu U) u^\dagger$

$\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u, \chi = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}$

$\Phi = \begin{pmatrix} \pi^0 + \eta \sqrt{2} & \pi^+ & K^+ \\ \pi^- \sqrt{2} - \eta \sqrt{6} & K^0 & -2\sqrt{6}\eta \\ K^- \sqrt{2} + \frac{\eta}{\sqrt{6}} & \bar{K}^0 & -2\sqrt{6}\eta \end{pmatrix}$

$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- - \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Xi^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$
S= − 1 meson-baryon sector is plenty of data

Good ground test for SU(3) chiral dynamics, where very strong SU(3) breaking effects due to the explicit presence of mesons/baryons with strangeness—Explicit breaking of chiral symmetry

Important isospin breaking effects due to cusps at thresholds, we work with the physical basis.
1) CROSS SECTIONS:

\[ K^- p \rightarrow K^- p, K^0 n, \pi^+ \Sigma^-, \pi^- \Sigma^+, \pi^0 \Sigma^0, \pi^0 \Lambda \]

In the fit we include data from threshold up to \( p_{lab} = 0.2 \) GeV.

2) Precisely Measured Ratios

\[
\gamma = \frac{\sigma(K^- p \rightarrow \pi^+ \Sigma^-)}{\sigma(K^- p \rightarrow \pi^- \Sigma^+)} = 2.36 \pm 0.04 ,
\]

\[
R_c = \frac{\sigma(K^- p \rightarrow \text{charged particles})}{\sigma(K^- p \rightarrow \text{all})} = 0.664 \pm 0.011 ,
\]

\[
R_n = \frac{\sigma(K^- p \rightarrow \pi^0 \Lambda)}{\sigma(K^- p \rightarrow \text{all neutral states})} = 0.189 \pm 0.015 ,
\]
5) WE ALSO CONSTRAINT OUR FITS CALCULATING AT O(p^2) IN PURE BARYON CHPT SEVERAL PION-NUCLEON OBSERVABLES, WHERE CHPT EXPANSION IS RELIABLE:

\[\begin{align*}
\sigma_{\pi N} &= -2m^2\pi (2b_0 + b_F + b_D), \\
a_{0+}^+ &= \frac{m^2\pi}{2\pi f^2} \left(-2b_1 + b_2 + b_3 - \frac{g_A^2}{8m}\right) [b_D, b_f \text{ and } b_3 \text{ in terms of } b_0, b_1 \text{ and } b_2], \\
m_0 &= m_p + 4m^2_K(b_0 + b_D - b_F) + 2m^2\pi(b_0 + 2b_F).
\end{align*}\]

\[\sigma_{\pi N} = 20, 30, 40 \text{ MeV (45±8 from Gasser, Leutwyler, Sainio PLB253,252 ('91), higher order corrections §10 MeV Gasser, AP254,192(‘97))}\]

\[m_0 = 0.7 \text{ or } 0.8 \text{ GeV}\]

\[a_{0+}^+ = (-1± 1) m_\pi 10^{-2} \text{ Exp. } -0.25± 0.49 \text{ Schröder et al., PLB469,25('99) and expected higher order corrections } +m_\pi 10^{-2} \text{ from unitarity Bernard et al. PLB309,421('93).}\]
1) RECENT FURTHER DATA INCLUDED IN THE EXTENDED ANALYSIS JAO EPJA28,63(2006)

6) $\sigma(K^-p \rightarrow \eta\Lambda)$ cross-section
On top of the $\Lambda(1670)$ resonance.

7) $\sigma(K^-p \rightarrow \Sigma^0\pi^0\pi^0)$
total cross-section and event distribution.

6) and 7) measured by the Crystall-Barrell Collaboration, 2001 and 2004, respectively. Precise experimental data.

8) $\Lambda\pi$ P- and S-wave phase shift difference at $\Xi^-$ mass $\delta_P - \delta_S = (4.6 \pm 1.4)^o$.

HyperCP Collaboration

155 points
For the calculation of the process $K^- p \rightarrow \pi^0 \pi^0 \Sigma^0$ we take as the production vertex the mechanism:

Which dominates due to the almost on-shell character of the intermediate proton.

The solid point means full $K^- p \rightarrow \pi^0 \Sigma^0$ S-wave

4. RESULTS

Two classes of fits $A, B$ with $1_s$ kaonic hydrogen $\Delta E$ and $\Gamma$:

A: Around DEAR (The fits are numerically more stable)
B: Away from DEAR.
Reproduction of the data by the new A-type fits (agree with DEAR) of JAO, EPJA28,63(’06)
### Experiment

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$2.36 \pm 0.04$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_c$</td>
<td>$0.664 \pm 0.011$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_n$</td>
<td>$0.189 \pm 0.015$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_E$</td>
<td>$193 \pm 38$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>$249 \pm 118$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_{\pi \Lambda}$</td>
<td>$4.6 \pm 2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table

<table>
<thead>
<tr>
<th>$\sigma_{\pi N}$</th>
<th>$20^\circ$</th>
<th>$30^\circ$</th>
<th>$40^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>2.36</td>
<td>2.36</td>
<td>2.37</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.629</td>
<td>0.628</td>
<td>0.628</td>
</tr>
<tr>
<td>$R_n$</td>
<td>0.168</td>
<td>0.171</td>
<td>0.173</td>
</tr>
<tr>
<td>$\Delta E$ (eV)</td>
<td>194</td>
<td>192</td>
<td>192</td>
</tr>
<tr>
<td>$\Gamma$ (eV)</td>
<td>324</td>
<td>302</td>
<td>270</td>
</tr>
<tr>
<td>$\Delta E_D$ (eV)</td>
<td>204</td>
<td>204</td>
<td>207</td>
</tr>
<tr>
<td>$\Gamma_D$ (eV)</td>
<td>361</td>
<td>338</td>
<td>305</td>
</tr>
<tr>
<td>$a_{K-p}$ (fm)</td>
<td>$-0.49 + i 0.44$</td>
<td>$-0.49 + i 0.41$</td>
<td>$-0.50 + i 0.37$</td>
</tr>
<tr>
<td>$a_0$ (fm)</td>
<td>$-1.07 + i 0.53$</td>
<td>$-1.04 + i 0.50$</td>
<td>$-1.02 + i 0.45$</td>
</tr>
<tr>
<td>$a_1$ (fm)</td>
<td>$0.44 + i 0.15$</td>
<td>$0.40 + i 0.15$</td>
<td>$0.33 + i 0.14$</td>
</tr>
<tr>
<td>$\delta_{\pi \Lambda}(\Xi)$ $(^\circ)$</td>
<td>3.4</td>
<td>4.5</td>
<td>5.7</td>
</tr>
<tr>
<td>$m_0$ (GeV)</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$a_{0+}^{+}(10^{-2} \cdot M^{-1})$</td>
<td>$-2.0$</td>
<td>$-2.2$</td>
<td>$-2.2$</td>
</tr>
</tbody>
</table>

### Units Table

<table>
<thead>
<tr>
<th>Units</th>
<th>$\sigma_{\pi N}$ MeV</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>MeV $^{-1}$</td>
<td>$f$</td>
<td>75.2</td>
<td>71.8</td>
<td>67.8</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$b_0$</td>
<td>$-0.615$</td>
<td>$-0.750$</td>
<td>$-0.884$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$b_D$</td>
<td>$+0.818$</td>
<td>$+0.848$</td>
<td>$+0.873$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$b_F$</td>
<td>$-0.114$</td>
<td>$-0.130$</td>
<td>$-0.138$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$b_1$</td>
<td>$+0.660$</td>
<td>$+0.670$</td>
<td>$+0.676$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$b_2$</td>
<td>$+1.144$</td>
<td>$+1.169$</td>
<td>$+1.189$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$b_3$</td>
<td>$-0.297$</td>
<td>$-0.316$</td>
<td>$-0.315$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$b_4$</td>
<td>$-1.048$</td>
<td>$-1.181$</td>
<td>$-1.307$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$a_1$</td>
<td>$-1.786$</td>
<td>$-1.591$</td>
<td>$-1.413$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$a_2$</td>
<td>$-0.519$</td>
<td>$-0.451$</td>
<td>$-0.386$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$a_5$</td>
<td>$-1.185$</td>
<td>$-1.170$</td>
<td>$-1.156$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$a_7$</td>
<td>$-5.251$</td>
<td>$-5.209$</td>
<td>$-5.123$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$a_8$</td>
<td>$-1.316$</td>
<td>$-1.310$</td>
<td>$-1.308$</td>
</tr>
<tr>
<td>GeV $^{-1}$</td>
<td>$a_9$</td>
<td>$-1.186$</td>
<td>$-1.132$</td>
<td>$-1.050$</td>
</tr>
</tbody>
</table>

**Three b’s are fixed in terms of the others from the $O(p^2)$ constraints**
These fits agree with all the experimental data, both scattering and atomic, including the most recent ones.

<table>
<thead>
<tr>
<th>Units</th>
<th>$\sigma_{\pi N}$ [MeV]</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>MeV</td>
<td>$f$</td>
<td>75.2</td>
<td>71.8</td>
<td>67.8</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$b_0$</td>
<td>-0.615</td>
<td>-0.750</td>
<td>-0.884</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$b_P$</td>
<td>+0.818</td>
<td>+0.848</td>
<td>+0.873</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$b_F$</td>
<td>-0.114</td>
<td>-0.130</td>
<td>-0.138</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$b_1$</td>
<td>+0.660</td>
<td>+0.670</td>
<td>+0.676</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$b_2$</td>
<td>+1.144</td>
<td>+1.169</td>
<td>+1.189</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$b_3$</td>
<td>-0.297</td>
<td>-0.316</td>
<td>-0.315</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$b_4$</td>
<td>-1.048</td>
<td>-1.181</td>
<td>-1.307</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$a_1$</td>
<td>-1.786</td>
<td>-1.591</td>
<td>-1.413</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$a_2$</td>
<td>-0.519</td>
<td>-0.454</td>
<td>-0.386</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$a_5$</td>
<td>-1.185</td>
<td>-1.170</td>
<td>-1.156</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$a_7$</td>
<td>-5.254</td>
<td>-5.209</td>
<td>-5.123</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$a_8$</td>
<td>-1.316</td>
<td>-1.310</td>
<td>-1.308</td>
</tr>
<tr>
<td>GeV^{-1}</td>
<td>$a_9$</td>
<td>-1.186</td>
<td>-1.132</td>
<td>-1.050</td>
</tr>
</tbody>
</table>

**Experiment**

- $\gamma$ \(2.36 \pm 0.04\)
- $R_c$ \(0.664 \pm 0.011\)
- $R_n$ \(0.189 \pm 0.015\)
- $\Delta E$ \(193 \pm 38\)
- $\Gamma$ \(249 \pm 118\)
- $\delta_{\pi\Lambda}$ \(4.6 \pm 2\)
Reproduction of the data by the new B-type fits (do not agree with DEAR) of JAO, EPJA28,63(’06)
Numerically it is simpler to obtain A-type fits.

The scattering length $a_{K^-p}$ is much larger than in the A-type fits.

These fits disagree with DEAR but agree with KEK.

Three b’s are fixed in terms of the others from the $O(p^2)$ constraints.
**K- p Scattering Length:**

Martin, NPB179,33(’81): $a_{K-p} = -0.67 + i0.64$ fm

Kaiser,Siegel,Weise, NPA594,325(’95): $a_{K-p} = -0.97 + i1.1$ fm

Oset,Ramos, NPA635,99(’98): $a_{K-p} = -0.99 + i0.97$ fm

Meissner,JAO PLB500,263(’01): $a_{K-p} = -0.75 + i1.2$ fm

Borasoy,Nissler,Weise,PRL94,213401(’05), EPJA25,79(’05): $a_{K-p} = -0.51 + i0.82$ fm They Cannot reproduce the elastic K- p ! K- p cross sections nor the DEAR measurement (compromise)

**Our works** $A_4^+ : a_{K-p} = -0.51 + i 0.42$ fm ; $B_4^+ : -1.01 + i 0.80$ fm

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{\pi N}$</th>
<th>20*</th>
<th>30*</th>
<th>40*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{K-p}$ (fm)</td>
<td>$-0.49 + i 0.44$</td>
<td>$-0.49 + i 0.41$</td>
<td>$-0.50 + i 0.37$</td>
<td></td>
</tr>
<tr>
<td>$a_0$ (fm)</td>
<td>$-1.07 + i 0.53$</td>
<td>$-1.04 + i 0.50$</td>
<td>$-1.02 + i 0.45$</td>
<td></td>
</tr>
<tr>
<td>$a_1$ (fm)</td>
<td>$0.44 + i 0.15$</td>
<td>$0.40 + i 0.15$</td>
<td>$0.33 + i 0.14$</td>
<td></td>
</tr>
</tbody>
</table>

**New A-type:**

<table>
<thead>
<tr>
<th></th>
<th>$a_{K-p}$ (fm)</th>
<th>$-1.01 + i 1.03$</th>
<th>$-0.93 + i 1.07$</th>
<th>$-1.06 + i 1.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$ (fm)</td>
<td>$-1.75 + i 1.15$</td>
<td>$-1.65 + i 1.30$</td>
<td>$-1.79 + i 1.10$</td>
<td></td>
</tr>
<tr>
<td>$a_1$ (fm)</td>
<td>$-0.13 + i 0.39$</td>
<td>$-0.14 + i 0.36$</td>
<td>$-0.12 + i 0.46$</td>
<td></td>
</tr>
</tbody>
</table>
5. SPECTROSCOPY

$$T_{ij} = \lim_{s \to s_R} \frac{\gamma_i \gamma_j}{s - s_R}$$

Residues

Pole Position $'(M_R - i\Gamma_R/2)^2$

Physical Riemann Sheets

<table>
<thead>
<tr>
<th>πΛ</th>
<th>πΣ</th>
<th>(\bar{K}N)</th>
<th>1.25</th>
<th>1.33</th>
<th>1.43</th>
<th>1.66</th>
<th>1.74</th>
<th>1.81</th>
<th>(\text{GeV})</th>
</tr>
</thead>
</table>

Different Riemann Sheets:

1RS 2RS 3RS 4RS 5RS 6RS

\(\eta \Lambda \) \(\eta \Sigma \) \(K \Xi \)
**Fit I: New A-type fit with $\sigma_{\pi N}=40$ MeV**

**I=0 Poles (MeV)**

| Pole   | Re(Pole) | -Im(Pole) | Sheet | $|\gamma_{\pi\Lambda}|$ | $|\gamma_{\pi\Sigma}|$ | $|\gamma_{\pi\Sigma}|_0$ | $|\gamma_{\pi\Sigma}|_1$ | $|\gamma_{\bar{K}N}|_0$ | $|\gamma_{\bar{K}N}|_1$ | $|\gamma_{\eta\Lambda}|$ | $|\gamma_{\eta\Sigma}|$ | $|\gamma_{K\Xi}|_0$ | $|\gamma_{K\Xi}|_1$ |
|--------|----------|-----------|-------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\Lambda(1310)$ | 1301 | 13 | 1RS | 0.03 | 1.12 | 0.02 | 0.01 | 5.83 | 0.05 | 0.41 | 0.04 | 2.11 | 0.03 |
| $\Lambda(1405)$ | 1309 | 13 | 2RS | 0.02 | 3.66 | 0.02 | 0.02 | 4.46 | 0.04 | 0.21 | 0.04 | 3.05 | 0.03 |
| $\Lambda(1414)$ | 1414 | 23 | 2RS | 0.14 | 4.24 | 0.13 | 0.01 | 4.87 | 0.39 | 0.85 | 0.20 | 9.35 | 0.11 |
| $\Lambda(1388)$ | 1388 | 17 | 3RS | 0.02 | 3.81 | 0.02 | 0.02 | 1.33 | 0.04 | 0.42 | 0.04 | 9.55 | 0.04 |
| $\Lambda(1670)$ | 1676 | 10 | 3RS | 0.01 | 1.28 | 0.03 | 0.00 | 1.67 | 0.01 | 2.19 | 0.07 | 5.29 | 0.07 |
| $\Lambda(1673)$ | 1673 | 18 | 4RS | 0.01 | 1.26 | 0.02 | 0.00 | 1.82 | 0.01 | 2.13 | 0.06 | 5.32 | 0.06 |
| $\Lambda(1800)$ | 1825 | 49 | 5RS | 0.02 | 2.29 | 0.02 | 0.00 | 2.10 | 0.02 | 0.89 | 0.03 | 7.43 | 0.09 |

**PDG:**
- $M=1405.5 \pm 4$ MeV
- $\Gamma=50 \pm 2$ MeV
- $M=1670 \pm 10$ MeV
- $\Gamma=35 \pm 15$ MeV
- $M=1700-1900$ MeV
- $\Gamma=65 - 600$ MeV
Assymetry in the width, before the $\eta\Lambda$ threshold $\Gamma=20$ MeV and above $\Gamma=36$ MeV.
### I=1 Poles (MeV)

| Re(Pole) | -Im(Pole) | Sheet | $|\gamma_{\pi}A|$ | $|\gamma_{\pi}\Sigma|$ | $|\gamma_{\pi}\Sigma|$ | $|\gamma_{\bar{K}}N|$ | $|\gamma_{K}N|$ | $|\gamma_{\eta}A|$ | $|\gamma_{\eta}\Sigma|$ | $|\gamma_{K}\Xi|$ | $|\gamma_{K}\Xi|$ |
|----------|-----------|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1425     | 6.5       | 2RS   | 1.35           | 0.24           | 1.66           | 0.01           | 0.35           | 3.92           | 0.05           | 4.23           | 0.49           | 2.98           |
| 1468     | 13        | 2RS   | 2.80           | 0.16           | 5.96           | 0.02           | 0.23           | 8.74           | 0.04           | 10.66          | 0.19           | 2.48           |
| 1433     | 3.7       | 3RS   | 0.65           | 0.08           | 0.80           | 0.00           | 0.12           | 1.58           | 0.02           | 5.82           | 0.20           | 2.14           |
| 1720     | 18        | 4RS   | 1.82           | 0.02           | 1.21           | 0.00           | 0.02           | 0.95           | 0.02           | 6.78           | 0.05           | 5.31           |
| 1769     | 96        | 6RS   | 2.65           | 0.00           | 0.61           | 0.00           | 0.00           | 2.48           | 0.00           | 3.32           | 0.01           | 4.22           |
| 1340     | 143       | 3-4RS | 1.33           | 0.14           | 5.50           | 0.02           | 0.02           | 1.58           | 0.00           | 3.28           | 0.03           | 1.20           |
| 1395     | 311       | 3-4RS | 2.08           | 0.01           | 1.49           | 0.01           | 0.00           | 1.24           | 0.00           | 7.63           | 0.01           | 3.97           |

**Σ(1750)**

PDG: M=1730-1800

$\Gamma=50 - 160$
On the physical axis between 1.4 and 1.5 GeV

Σ(1480) Bumps * in PDG. Observation by COSY PRL96,012002(’06)

1.5 GeV

20 MeV

| πΛ | | πΣ | | ηΣ |
|---|---|---|---|---|---|---|---|
| 1.425 | 6.5 | 2RS |
| 1.35 | 0.24 | 1.66 | 0.01 | 0.35 | 3.92 | 0.5 | 4.23 | 0.49 | 2.98 |
| 1.468 | 13 | 2RS |
| 2.80 | 0.16 | 5.96 | 0.02 | 0.23 | 8.74 | 0.04 | 10.66 | 0.19 | 2.48 |
| 1.433 | 3.7 | 3RS |
| 0.65 | 0.08 | 0.80 | 0.00 | 0.12 | 1.58 | 0.02 | 5.82 | 0.20 | 2.14 |

πΣ 2

ηΣ 2
For the open channels $\pi\Lambda$, $\pi\Sigma$, $\bar{K}N$ it is a distorted bump

For the closed channels $\eta\Sigma$ and $K\Xi$ it is a clear resonance shape
The amplitudes show a broad bump after the $\bar{K}N$ threshold and before that of the $\eta\Sigma$

Multipole interference effect

$\Sigma(1620)$

Physical Axis

$I=1 \mid \pi\Sigma \mid^2$

$\mid \pi\Lambda \mid^2$
I=2 Pole (MeV) at 1722-i 181 MeV

Exotic state

The only resonance in I=2

Non uniform shape for $\pi\Sigma$. I=2 is of size not negligible small compared with the other $\pi\Sigma$ isospin channels.
Fit I: New A-type fit with $\sigma_{\pi N}=40$ MeV

$I=0$: $\Lambda(1305), \Lambda(1405), \Lambda(1670), \Lambda(1800)$

$I=1$: $\Sigma(1480), \Sigma(1620), \Sigma(1750)$

Fit II: New B-type fit with $\sigma_{\pi N}=40$ MeV

$I=0$: $\Lambda(1350), \Lambda(1405), \Lambda(1670), \Lambda(1800)$

$I=1$: $\Sigma(1480), \Sigma(1620), \Sigma(1750)$

Only in $KN$

$8-\bar{8}=1\otimes8_s\otimes8_a\otimes10\otimes10\otimes27$

Fit I: has attractive SU(3) kernels for 1, 8_s, 8_a, 27. This can accommodate 4 I=0 and 3 I=1 resonances.

Fit II: has attractive SU(3) kernels for 1, 8_s, 8_a, 10. This can accommodate 3 I=0 and 3 I=1 resonances.
4. CONCLUSIONS

1. A UCHPT study of meson-baryon dynamics with strangeness=-1 in S-wave up to NLO or O(p^2)

2. We reproduce simultaneously scattering data, including the recent and precise results from the Crystall Ball Collaboration, and atomic data on kaonic hydrogen given by the DEAR Collaboration with the A-type fits.

3. We also find other fits, B-type, that do not reproduce DEAR, but agree with KEK and with scattering data.

4. The A-type fits are also able to generate the four Λ’s: Λ(1310), Λ(1405), Λ(1670), Λ(1800). All the ones quoted in PDG with 1/2⁻ in this energy range.

5. These fits also generate the I=1 resonances quoted in the PDG: Σ(1440), Σ(1620) and Σ(1750). 5+6 is a UNIQUE feature in the literature.

6. The B-type fits are not able to generate a comparable set of resonances. The Λ(1800) and the Σ(1750) are missing.
7. Fits A are then preferred over Fits B.

8. \(a_{K-p} = -0.5 + i 0.4 \text{ fm}\)
Threshold

The only place where Wigner bound is somewhat violated but there cannot be applied because the phase is not differentiable – inelastic cusp

Reply to Borasoy, Nissler, Weise
Comment PRL96, 199201(’06)
Figure 1: First panel is boric hydrogen strong energy shift and width. In the rest, the solid lines correspond to the fit $A_4^+$ and the dashed ones to $B_4^+$. For further details see the text.
$\pi\Sigma$ I=0 Mass Distribution

Typically one takes: 

$$\frac{dN_{\pi\Sigma}}{dE} = C |T^{I=0}_{\pi\Sigma \rightarrow \pi\Sigma}|^2 p_{\pi\Sigma}$$

As if the process were elastic

E.g: Dalitz, Deloff, JPG 17,289 (’91); Müller, Holinde, Speth NPA513,557(‘90), Kaiser, Siegel, Weise NPB594,325 (’95); Oset, Ramos NPA635, 99 (’89)

But the $\bar{K}N$ threshold is only 100 MeV above the $\pi\Sigma$ one, comparable with the widths of the present resonances in this region and with the width of the shown invariant mass distribution. This prescription is ambiguous, why not?

$$\frac{dN_{\pi\Sigma}}{dE} = C |T^{I=0}_{\bar{K}N \rightarrow \pi\Sigma}|^2 p_{\pi\Sigma}$$

We follow the Production Process scheme previously shown: already employed for this case in Meissner, JAO PLB500,263(’01)

$$F = (I + R \cdot g)^{-1} \cdot \xi , \quad \xi^T = (0, r_1, r_1, r_1, r_2, r_2, 0, 0, 0, 0)$$

$$\frac{r_2}{r_1} = -0.28$$

I=0 Source

$r_2=0$ (previous approach)
$\delta_P - \delta_S \Lambda \pi$ PHASE SHIFTS DIFFERENCE
AT THE $\Xi^-$ MASS, RECENT MEASUREMENTS
FROM THE DECAY PARAMETERS $\Xi^- \rightarrow \Lambda \pi^-$:

$(4.6 \pm 1.4 \pm 1.2)^0$ Huang et al. (HyperCP Coll.) PRL93,011802 ('04)
$(3.2 \pm 5.3 \pm 0.7)^0$ Chakravorty et al. (E756 Coll.) PRL91,031601 ('03)

**Fit $A_4^+$ PREDICTS: 2.5^0** COMPATIBLE WITH DATA

For Fit $B_4^+: 0.2^0$
\[ |T^{I=2}_{\pi\Sigma}|^2 \]

\[ \text{RESONANCE SIGNAL} \]

\[ \text{DESTRUCTIVE INTERFERENCE WITH BACKGROUND} \]

GeV

\[ f_0(980) \]
INFLUENCE OF THE I=1 RESONANCES IN $\pi\Sigma$ EVENT DISTRIBUTION

$\gamma p \rightarrow K^+ \Lambda(1405) K^+ \pi^+\Sigma^- , \pi^- \Sigma^+$

J.K. Ahn, NP A721 (’03) 715c

LINE:
Nacher, Oset, Toki, Ramos PL B455 (’99)55
The set of Feynman diagrams contributing to the energy shift of the kaonic hydrogen up-to-and-including $\mathcal{O} (\alpha^4, \alpha^3 (m_d - m_u))$. Solid, dashed, double, dotted, wiggly and spring lines correspond to the proton, $K^-$, neutron, $\bar{K}^0$, Coulomb and transverse photons, respectively. The electrons run in the closed loops shown in diagrams (d) and (i). The diagrams (f) and (i) contain Coulomb ladders – the contributions with 0, 1, 2, $\cdots$ Coulomb photons exchanged.
Comparison of New and Old Data:

Predictions of the ground-state strong shift $\Delta E_1^s$ and width $\Gamma_1$. Filled circles correspond to using the original Deser formula, empty circles to using $T_{K_N}^{(0)}$ instead of $\frac{1}{2}(a_0 + a_1)$ in this formula and filled boxes to our final formula with $\delta T_{K_N} = \delta_{\nu N} = 0$.


$$a_0 = -1.31 + 1.24i \quad ; \quad a_1 = 0.26 + 0.66i$$


$$a_0 = -1.70 + 0.68i \quad ; \quad a_1 = 0.37 + 0.60i$$

DEAR  \[ X = 0 \rightarrow T_{Kp}^{th} = (-0.33 \pm 0.10) + i(0.28 \pm 0.10) \text{ fm} \]

M.Iwasaki et al. PRL78(1997)3067  \[ X = 0 \rightarrow T_{Kp}^{th} = (-0.78 \pm 0.15) + i(0.50 \pm 0. \]
Reproduction of the data by the fits of Prades, Verbeni, JAO PRL95(’05), plus an O(p) fit.

Three b’s are fixed in terms of the others from the $O(p^2)$ constraints

$$\sigma_{\pi N} = 40 \text{ MeV}$$

$$m_0 = 0.8 \text{ GeV}$$

$$a_{0+}^+ = (-1 \pm 1)m_{\pi}^{-1}10^{-2}$$

$$a_2 = a_3 = a_4, \quad a_5 = a_6, \quad a_9 = a_{10}$$
Fit A reproduces simultaneously scattering data plus DEAR measurement. It was the first chiral fit to accomplish this. However, it fails to reproduce the Crystall Ball data.
Reproduction of the data by the fits of Prades, Verbeni, JAO PRL95(’05), plus an O(p) fit.

**Solid**: Fit A.

**Dashed**: Fit B.

**Dash-Dotted**: O(p) Fit.