Outline

- Neutron stars
- (Ultrafast) Rotation
- Compressed baryonic matter in rotating neutron stars
- Hyperons and quarks in neutron stars
- Gravitational radiation reaction driven instabilities
- Summary
Isolated Rotating Neutron Stars (Pulsars)
Neutron Stars in X-ray Binaries (e.g. LMXBs)

XMM Newton
A few most intriguing facts about neutron stars:

- $N \sim 10^{57}$ baryons
- $M \sim 1 - 2 \ M_{\text{sun}}$
- $R \sim 10 - 12 \ km$
- $B \sim 10^{8 - 16} \ G$, $E \sim 10^{14 - 18} \ V/cm$
- $T \sim 10^{6 - 11} \ K$
- Total number $\sim 10^9 - 10^{10}$
- Currently known $\sim 1600$

Latest discovery: PSR in Terzan 5 $P_{J1748-2446ad} = 1.39 \ ms \ (716 \ Hz)$ !

$P_{B1937+21} = 1.58 \ ms \ (630 \ Hz)$

Accretion rates: $10^{-8}$ to $10^{-10} \ M_{\text{sun}}/yr$
"Neutron" Star Composition in 2006

Einstein's Field Equations for Rotating Compact Objects

- Metric: \( ds^2 = -e^{2\nu} \, dt^2 + e^{2\Lambda} \, dr^2 + r^2 \, d\Omega^2 \)

- Christoffel symbols:
\[
\Gamma^\sigma_{\mu\nu} = g^{\sigma\lambda} \left( \partial_\nu g_{\mu\lambda} + \partial_\mu g_{\nu\lambda} - \partial_\lambda g_{\mu\nu} \right) / 2
\]

- Riemann tensor:
\[
R^\tau_{\mu\nu\sigma} = \partial_\nu \Gamma^\tau_{\mu\sigma} - \partial_\sigma \Gamma^\tau_{\mu\nu} + \Gamma^\kappa_{\mu\sigma} \Gamma^\tau_{\kappa\nu} - \Gamma^\kappa_{\mu\nu} \Gamma^\tau_{\kappa\sigma}
\]

- Ricci tensor: \( R_{\mu\nu} = R^\tau_{\mu\sigma\nu} \, g^{\sigma\tau} \)

- Scalar curvature: \( R = R_{\mu\nu} \, g^{\mu\nu} \)

- Einstein's equation:
\[
G^\mu^\nu = R^\mu^\nu - g^\mu^\nu / 2 = 8\pi \, T^\mu^\nu (\varepsilon, P) \quad \Rightarrow
\]
\[
\frac{dP}{dr} = - \frac{m \left( 1 + 4\pi \, r^3 \, P / m \right)}{r^2 \left( 1 - 2m / r \right)} \left[ 1 + \frac{P}{\varepsilon} \right]
\]
Tolman-Oppenheimer-Volkoff
Einstein's Field Equations for Rotating Compact Objects

- Metric: \[ ds^2 = -e^{2\nu} \, dt^2 + e^{2(\alpha + \beta)} \, r^2 \sin^2 \theta \left( d\phi - N^\phi \, dt \right)^2 + e^{2(\alpha - \beta)} \left( dr^2 + r^2 \, d\theta^2 \right) \]

- Christoffel symbols:
  \[ \Gamma^\sigma_{\mu\nu} = g^{\sigma\lambda} \left( \partial_\nu g_{\mu\lambda} + \partial_\mu g_{\nu\lambda} - \partial_\lambda g_{\mu\nu} \right) / 2 \]

- Riemann tensor:
  \[ R^\tau_{\mu\nu\sigma} = \partial_\nu \Gamma^\tau_{\mu\sigma} - \partial_\sigma \Gamma^\tau_{\mu\nu} + \Gamma^\kappa_{\mu\sigma} \Gamma^\tau_{\kappa\nu} - \Gamma^\kappa_{\mu\nu} \Gamma^\tau_{\kappa\sigma} \]

- Ricci tensor: \[ R_{\mu\nu} = R^\tau_{\mu\sigma\nu} \, g^\sigma_{\tau} \]

- Scalar curvature: \[ R = R_{\mu\nu} \, g^{\mu\nu} \]

- Limiting frequency? 1. Kepler frequency, \[ \Omega^K = r^{-1} \, e^{\nu - \alpha - \beta} \, U^K + N^\phi \]
  2. GRR driven instabilities

- Differential rotation/uniform rotation?
Differentially Rotating Stellar Objects

- $M = 1.4 M_{\text{sun}}$
- $\nu_{eq} = 290 \text{ Hz}$
- $\nu_c = 140 \nu_{eq}$

Open issue: dynamical stability; secular stability

Mixed phase of baryons and qarks
Pulsar's Composition depends on Spin Frequency!


60% change!
Model Quark-Hadron Composition
Quark-hybrid star losing its quark matter core due to changes in the rotation rate

Chubarian, Grigorian, Poghosyan, Blaschke A&A 357 (2000) 968;
Backbending - well known in nuclear physics ... and from the Olympics

Model Composition of a $M=1.7 \, M_{\text{sun}}$

$R_{\text{eq}} = 11.5 \, \text{km}$, $\varepsilon_c / \varepsilon_0 = 6.5 \, @ \, \Omega = 0$

Equation of state:
DD RBHF, Bonn, Hofmann, Keil, Lenske.
PRC 64 (2001) 034314

P. Rosenfield, SDSU, 2006
Gravitational-Radiation Reaction Driven Instabilities …

Driven by rotation at $\nu > \nu_{\text{critical}}$

Damped by
- Shear viscosity
- Bulk viscosity
Gravitational-Radiation Reaction Driven Instabilities ...

Star rotates at $\Omega$

gravitational radiation

Perturbation rotates at $\omega_m(\Omega)$

quadrupole mode

\[ A \propto A_0 e^{-i \omega_m(\Omega)t + i m \phi - t/\tau(\Omega)} \]
The underlying theory...

\[ \frac{\partial \varepsilon}{\partial t} + \nabla_i (\varepsilon v^i) = 0 \]
\[ \varepsilon \left( \frac{\partial v^i}{\partial t} + v^j \nabla_j v^i \right) = -\nabla^i P + \varepsilon \nabla^i \phi \]
\[ \nabla^i \nabla_i \phi = -4 \pi \varepsilon \]

Energy contained in perturbation:
\[ \frac{dE}{dt} = -\int d^3 x \left( 2 \eta \delta \sigma^{ij} \delta \sigma_{ij} + \zeta \delta \sigma \delta \sigma - \bar{\omega} \sum_l N_l \omega^{2l+1} \delta D_L^m \delta \bar{D}_l^m \right) \]

Ansatz for amplitudes:
\[ A \propto A_0 e^{-i \omega(\Omega) t + i m \phi - t/\tau(\Omega)} \]

\[ \Rightarrow \quad -\frac{1}{2 E} \frac{dE}{dt} = \frac{1}{\tau} = \frac{1}{\tau_\xi} + \frac{1}{\tau_\eta} + \frac{1}{\tau_{GR}} \]

Final equation:
\[ 0 \equiv \frac{1}{\tau} = \frac{1}{\tau_\xi} + \frac{1}{\tau_\eta} + \frac{1}{\tau_{GR}} \quad \text{If } 1/\tau > 0: \text{ perturbation is damped} \]
\[ \text{If } 1/\tau < 0: \text{ perturbation grows} \]
Gravitational-Radiation Reaction Driven Instabilities ...
Critical Angular Velocity of Rotating Neutron Stars

f-mode instability

unstable rotation

r-mode instability

unstable rotation

From Lindblom & Owen. PRD 65 (2002) 063006

Summary

- Cores of neutron stars are likely to contain significant amount of strangeness
  - confined in hyperons
  - unconfined quark matter
- Strangeness content will change dramatically with neutron star frequency.
- Backbending of isolated neutron stars
- $r$-modes damped by hyperons
- Need more neutron star data (SkA)
The "Square-Kilometre-Array"

... an observational windows on inner workings of neutron stars

- Sensitivity ~100 times higher than the VLA sensitivity
- ~ 20,000 pulsars expected to be discovered
- ~ 1000 MSPs
- Pulsars around black holes
- Testing GR
- Initial operations start ~2016
- Final operations start ~2020
Interacting neutron gas fails(!), and so does a non-interacting neutron gas!!
Overview of Sources

- Neutron Star & Black Hole Binaries
  - inspiral
  - merger
- Spinning NS’s
  - LMXBs
  - Pulsars
- NS Birth (SN, AIC)
  - tumbling
  - convection
- Stochastic background
  - big bang
  - early universe

Source: J. Hartle