Study of exotic hadrons in s-wave scatterings induced by chiral interaction in the flavor symmetric limit

Tetsuo Hyodo\textsuperscript{a}

D. Jido\textsuperscript{a}, and A. Hosaka\textsuperscript{b}

\textit{YITP, Kyoto}\textsuperscript{a} \quad \textit{RCNP, Osaka}\textsuperscript{b}

2006, Oct. 13th
Exotic hadrons: states other than $q\bar{q}$, $qqq$.

---> QCD *does not forbid* exotic states, effective models neither.

Experimentally, (almost?) completely absent
---> highly non-trivial fact

Existence of exotic hadrons?

- Resonance saturation + duality
  J.L. Rosner, Phys. Rev. Lett. 21, 950 (1968)

- Large $N_c$ + heavy quark

---> Examine existence of exotic hadrons in flavor SU(3) limit.
Hadron-NG boson bound state

Chiral symmetry

s-wave low energy interaction

\[ V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \]

\[ C_{\text{exotic}} = 1 \]

Scattering theory

Critical strength for a bound state

\[ C_{\text{crit}} = \frac{2f^2}{m(-G(M_T + m))} \]

physical values:

\[ C_{\text{exotic}} < C_{\text{crit}} \]

No exotic state exists.
Chiral symmetry

Low energy s-wave interaction

Scattering of a target (T) with the pion (Ad)

\[
\alpha \left[ \frac{\text{Ad}(q)}{T(p)} \right] = \frac{1}{f^2} \frac{p \cdot q}{2M_T} \langle F_T \cdot F_{\text{Ad}} \rangle_\alpha + \mathcal{O} \left( \left( \frac{m}{M_T} \right)^2 \right)
\]

In s-wave,

\[
V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T}
\]

- proportional to pion energy
- pion decay constant (No LEC)


\[
C_{\alpha,T} \equiv -\langle 2F_T \cdot F_{\text{Ad}} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3 \quad \text{(for } N_f = 3)\]
### Coupling strengths: Examples

#### Examples of $C_\alpha$ : (positive is attractive)

$$C_{\alpha,T} = C_2(T) - C_2(\alpha) + 3$$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>1</th>
<th>8</th>
<th>10</th>
<th>10</th>
<th>27</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T=8\ (N,\Lambda,\Sigma,\Xi)$</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>$-2$</td>
<td></td>
</tr>
<tr>
<td>$T=10(\Delta,\Sigma^<em>,\Xi^</em>,\Omega)$</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>$-3$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>3</th>
<th>6</th>
<th>15</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T=3\ (\Lambda_c,\Xi_c)$</td>
<td>3</td>
<td>1</td>
<td>$-1$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$T=6\ (\Sigma_c,\Xi_c^*,\Omega_c)$</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

- **Exotic channels**: mostly repulsive
- **Attractive interaction**: $C = 1$
## Coupling strengths: General expression

\( T = [p, q] \quad \alpha \in [p, q] \otimes [1, 1] \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( C_{\alpha, T} )</th>
<th>sign</th>
<th>( \Delta E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>([p + 1, q + 1])</td>
<td>(-p - q)</td>
<td>repulsive</td>
<td>(1 \text{ or } 0)</td>
</tr>
<tr>
<td>([p + 2, q - 1])</td>
<td>(1 - p)</td>
<td></td>
<td>(1 \text{ or } 0)</td>
</tr>
<tr>
<td>([p - 1, q + 2])</td>
<td>(1 - q)</td>
<td></td>
<td>(1 \text{ or } 0)</td>
</tr>
<tr>
<td>([p, q])</td>
<td>3</td>
<td>attractive</td>
<td>0</td>
</tr>
<tr>
<td>([p, q])</td>
<td>3</td>
<td>attractive</td>
<td>0</td>
</tr>
<tr>
<td>([p + 1, q - 2])</td>
<td>(3 + q)</td>
<td>attractive</td>
<td>0 or (-1)</td>
</tr>
<tr>
<td>([p - 2, q + 1])</td>
<td>(3 + p)</td>
<td>attractive</td>
<td>0 or (-1)</td>
</tr>
<tr>
<td>([p - 1, q - 1])</td>
<td>(4 + p + q)</td>
<td>attractive</td>
<td>0 or (-1)</td>
</tr>
</tbody>
</table>

- C should be integer.
- Sign is determined for most cases.
Chiral symmetry

Exoticness: minimal number of extra $\bar{q}q$.

For $[p, q]$ and baryon number $B$,

$$E = \epsilon \theta(\epsilon) + \nu \theta(\nu)$$

$$\epsilon \equiv \frac{p + 2q}{3} - B, \quad \nu \equiv \frac{p - q}{3} - B$$

$\Delta E = E_\alpha - E_T = +1$ is realized when

1. $\Delta \epsilon = 1, \Delta \nu = 0, \epsilon_T \geq 0$,
   $$\alpha = [p + 1, q + 1] : C_{\alpha, T} = -p - q \quad \text{repulsive}$$

2. $\Delta \epsilon = 0, \Delta \nu = 1, \nu_T \geq 0$,
   $$\alpha = [p + 2, q - 1] : C_{\alpha, T} = 1 - p$$
   attraction: $p = 0$ then $\nu_T \geq 0 \rightarrow B \leq -q/3$ not considered here

3. $\Delta \epsilon = 1, \Delta \nu = -1, \nu_T \leq 0$,
   $$\alpha = [p - 1, q + 2] : C_{\alpha, T} = 1 - q$$
   attraction: $q = 0$ then $\nu_T \leq 0 \rightarrow B \geq p/3 \quad \text{OK!}$

Universal attraction for more “exotic” channel

$C_{\text{exotic}} = 1$ for $T = [p, 0], \quad \alpha = [p - 1, 2]$
Chiral symmetry

Coupling strengths: large $N_c$ behavior

For arbitrary $N_c$, 

$$[p, q] \rightarrow \left[ p, q + \frac{3 - N_c}{2} \right]$$ 

$$V \propto -\frac{1}{f^2} C \sim \frac{1}{N_c} C(N_c)$$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$C^{\alpha,\alpha'}(N_c)$</th>
<th>$V(N_c \rightarrow \infty)$</th>
<th>$\Delta E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[p + 1, q + 1]$</td>
<td>$(3 - N_c)/2 - p - q$</td>
<td>repulsive</td>
<td>1 or 0</td>
</tr>
<tr>
<td>$[p + 2, q - 1]$</td>
<td>$1 - p$</td>
<td>0</td>
<td>1 or 0</td>
</tr>
<tr>
<td>$[p - 1, q + 2]$</td>
<td>$(5 - N_c)/2 - q$</td>
<td>repulsive</td>
<td>1 or 0</td>
</tr>
<tr>
<td>$[p, q]$</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$[p + 1, q - 2]$</td>
<td>$(3 + N_c)/2 + q$</td>
<td>attractive</td>
<td>0 or $-1$</td>
</tr>
<tr>
<td>$[p - 2, q + 1]$</td>
<td>$3 + p$</td>
<td>0</td>
<td>0 or $-1$</td>
</tr>
<tr>
<td>$[p - 1, q - 1]$</td>
<td>$(5 + N_c)/2 + p + q$</td>
<td>attractive</td>
<td>0 or $-1$</td>
</tr>
</tbody>
</table>

- Exotic attraction $\rightarrow$ repulsion
- No attraction in exotic channels.
Renormalization and bound states

Solve the scattering problem with \( V_\alpha = -\frac{\omega}{2f^2} C_{\alpha,T} \)

\[
T = \frac{1}{1 - VG} V
\]

Unitarity : OK

Renormalization parameter : condition

\( G(\mu) = 0, \quad \Leftrightarrow \quad T(\mu) = V(\mu) \quad \text{at} \quad \mu = M_T \)


Approximate crossing symmetry : OK

Bound state:

\[
1 - V(M_b)G(M_b) = 0 \quad M_T < M_b < M_T + m
\]
Scattering theory

Critical attraction

\[ 1 - V(\sqrt{s})G(\sqrt{s}) : \text{monotonically decreasing.} \]

Fixed

\[ G(M_T) = 0 \]
\[ 1 - VG = 1 \]

Critical attraction:

\[ 1 - VG = 0 \text{ at } \sqrt{s} = M_T + m \]

\[ C_{\text{crit}} = \frac{2f^2}{m(-G(M_T + m))} \]
Critical attraction and exotic channel

\[ m = 368 \text{ MeV} \text{ and } f = 93 \text{ MeV} \]

\[ C_{\text{exotic}} = 1 \]

Strength is not enough.
Summary

We study the exotic bound states in s-wave chiral dynamics in flavor SU(3) limit.

There are attractions in exotic channels, with universal and the smallest strength: $C_{\text{exotic}} = 1$.

This is not enough to generate a bound state: $C_{\text{exotic}} < C_{\text{crit}}$.

This does not exclude the exotic states with other origin.