Strange Dibaryon and KNN-\(\pi N\Sigma\) Coupled Channel

Y. Ikeda, T. Sato (Osaka University)

Table of contents

1. Motivation
2. Formalism (3-body equation)
3. Results (KNN resonance state)
4. Summary
1. Motivation

Kaon bound states in the few body system

✓ Theoretical study (Akaishi and Yamazaki)
  AY interaction
  -> Reproducing kaonic hydrogen and KN scattering data
  Kaon optical potential (g-Matrix method)
  \[ V_{\text{opt}} = -119 - i11 \text{ (MeV)} \]
  -> Indication of "Deeply bound kaonic nuclear states"

✓ Experimental study (FINUDA Collaboration)

KNN bound state
  Binding energy
  115 MeV
  Decay Width
  67 MeV

Kaon absorption?? (Magas et al.)
Our Investigation

✓ We investigate the possible [KNN]_{(I=1/2,J=0)} 3-body resonance state.

We consider s-wave 3-body KNN state. We expect most strong attractive interaction in this configure.

L=0 (s-wave interaction)

Coupled channel Faddeev equation

Find

KNN 3-body resonance
2. Formalism

Meson-Baryon interaction

Chiral effective Lagrangian

\[ \mathcal{L}_I = \frac{i}{8F_{\pi}^2} \text{Tr} [\bar{B} \gamma^\mu [\Gamma_\mu^2, B]] \]

\[ \Gamma_\mu^2 = \phi (\partial_\mu \phi) + (\partial_\mu \phi) \phi + \ldots \]

\[ \phi : \text{Meson field}, \ B : \text{Baryon field} \]

We employ the dipole form factor.

✓ Cut-off parameter

\[ \rightarrow \text{fit scattering length (Kaonic hydrogen, Martin's analysis)} \]
AGS equation for 3-body amplitude

\[ X_{ij} = (1 - \delta_{ij})Z_{ij}(W) + \sum_{n \neq i} Z_{in}(W)\tau_n(W)X_{nj}(W) \]

\[ X_{ij} = K, N, \pi + KN-\pi\Sigma, NN, \pi N \]

Eigenvalue equation for Fredholm kernel

\[ Z(W)\tau(W)|\phi_n(W)\rangle = \eta_n(W)|\phi_n(W)\rangle \]

\[ \eta_n(W_p) = 1 \quad \text{Pole of 3-body amplitude} \]

\[ W_p = B - i \Gamma / 2 \]

Similar to \( \pi NN, \eta NN, K\cdot d \) analyses. (Matsuyama, Yazaki, .......)
3. Numerical Results

How this attractive KN interaction contributes to 3-body binding system is determined by solving dynamical 3-body eq. directly!!

\[ t_i(W - E_i(p_i)) = v_i + v_i G_0(W) t_2(W - E_i(p_i)) \]

\( W_{KN} \text{ (MeV)} \)

\( \frac{T}{k} \text{ (fm)} \)

imaginary part

real part

attractive

repulsive

spectator energy

KN amplitude bellow threshold

KN threshold
The pole position of KNN 3-body resonance state

Non-relativistic

Relativistic

KNN threshold

KNN \rightarrow \text{physical energy plane}

\pi \Sigma N \rightarrow \text{unphys}

\begin{align*}
-44.9 - i21.0 \text{ (MeV)}
\end{align*}

\begin{align*}
-59.2 - i15.9 \text{ (MeV)}
\end{align*}

a: KN interaction only

b: KN interaction + NN interaction
4. Summary

- We solve 3-body equation directly.
- We can find the resonance pole in the KNN physical and $\pi \Sigma N$ unphysical energy plane.
  \[ W_{\text{pole}} = -59.2 - i15.9 \text{ MeV using relativistic kinematics} \]

Future works

- The pole position strongly depends on KN interaction.
  - form factor (dipole $\rightarrow$ monopole)
  - explicit resonance model (3-quark $\Lambda(1405)$)
- Effects of kaon absorption (p-wave interaction)
2-body Meson-Baryon Potential

**Chiral effective lagrangian**

\[
\mathcal{L}_I = \frac{i}{8 F^2 \pi} Tr [\bar{B} \gamma^\mu [\Gamma_\mu^2, B]]
\]

\[
\Gamma_\mu^2 = \phi (\partial_\mu \phi) + (\partial_\mu \phi) \phi
\]

φ: Meson field, B: Baryon field

**S-wave meson-baryon interaction**

\[
V_{MB}(\vec{q}', \vec{q}) = -\lambda^{(I)}_{\alpha \beta} \frac{1}{(2\pi)^3} \frac{1}{8 F^2 \pi} \sqrt{\frac{(E' + M)(E + M)}{2M 2M}} \frac{1}{\sqrt{2\omega' 2\omega}} \times (\omega' + \omega + E' + E - 2M) \left(\frac{\Lambda^2}{\vec{q}'^2 + \Lambda^2}\right)^2 \left(\frac{\Lambda^2}{\vec{q}^2 + \Lambda^2}\right)^2
\]

- coupling constant
- Dipole type form factor
The pole position of $\Lambda(1405)$ state

**a:** KN interaction only

**b:** KN interaction + $\pi\Sigma$ coupling $\times 0.5$

**c:** KN interaction + $\pi\Sigma$ full coupling

$M_R - i\Gamma_R/2 = 1411 - i22.5$ MeV

KN $\rightarrow$ physical energy plane

$\pi\Sigma$ $\rightarrow$ unphysical energy plane
Yamaguchi type NN interaction

\[ V_i(p, p') = g_i(p) g_i(p') - h_i(p) h_i(p') \]

\[ g_i(p) = C_R p^l / (p^2 + a_R^2)^{(l+2)/2} \]
\[ h_i(p) = C_A p^l / (p^2 + a_A^2)^{(l+2)/2} \]

We take an attractive term only.

Strong attraction!?
Faddeev eq. for T-matrix

\[ T_i(W) = t_i(W) + \sum_{j \neq i} t_i(W)G_0(W)T_j(W) \]
\[ t_i(W) = v_i + v_iG_0(W)t_i(W) \]

AGS eq for 3-body amplitude

\[ X_{ij} = (1 - \delta_{ij})Z_{ij}(W) + \sum_{n \neq i} Z_{in}(W)\tau_n(W)X_{nj}(W) \]
Formal solution of AGS equation

AGS equation

\[ X(W) = Z(W) + \frac{Z(W)\tau(W)X(W)}{1 - \eta_n(W)} \]

Fredholm type kernel

Eigenvalue equation for Fredholm kernel

\[ Z(W)\tau(W)\langle \phi_n(W)| = \eta_n(W)|\phi_n(W)\rangle \]

Formal solution for 3-boby amplitude

\[ X(W) = \sum_n \frac{|\phi_n(W)\rangle\langle \phi_n(W)|Z(W)}{1 - \eta_n(W)} \]

\[ \eta_n(W_p) = 1 \rightarrow \text{resonance state} \]
2-body scattering term \( \tau(W) \)

\[
\tau^{-1}_i(W - E_i(p_i)) = \frac{1}{\gamma_i} - \int dq_i \frac{g(q_i)^2}{W - E_i(p_i) - \sqrt{(E_j + E_k) + p_i^2}}
\]
1 particle exchange term $Z_{ij}(W)$

$\checkmark \int dp_j \frac{g(\vec{q}_i)g(\vec{q}_j)\gamma_j(W - E_j(\vec{p}_j))}{W - E_i(\vec{p}_i) - E_i(\vec{p}_i) - E_k(\vec{p}_k)} \phi_n(\vec{p}_j) = \eta_n(W)\phi_n(\vec{p}_i)$

$\rightarrow$ Logarithmically singularity