Search for a $K^-pp$ bound state

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**K-pp bound state**

FINUDA collaboration: evidence for a deeply bound state
(correlated $\Lambda$ and $p$, $E\sim 100$ MeV):

Another interpretation of the experiment:

G-matrix calculation:
(not explicit treatment of expected strong absorption effects)*

⇒ More rigorous 3-body calculation, treating properly
the few-body dynamical aspects is called for

Present calculation:
Non-relativistic 3-body Faddeev equations in AGS form,
many-channel modification
Usual AGS 3-body equations

\[ U_{11} = T_2 G_0 U_{21} + T_3 G_0 U_{31} \]

\[ U_{21} = G_0^{-1} + T_1 G_0 U_{11} + T_3 G_0 U_{31} \]

\[ U_{31} = G_0^{-1} + T_1 G_0 U_{11} + T_2 G_0 U_{21} \]

defines \( U_{ij} \) operators (i,j - complementary indexes and indexes of particles)

\[ U_{11} : \quad 1 + (23) \rightarrow 1 + (23) \]

\[ U_{21} : \quad 1 + (23) \rightarrow 2 + (31) \]

\[ U_{31} : \quad 1 + (23) \rightarrow 3 + (12) \]

\( KN \) interaction strongly connected with \( \pi \Sigma \) channel via \( \Lambda(1405) \) resonance

\( \Rightarrow \) many-channel equations are needed

Numeration of particles and channels for KNN-\( \pi \Sigma N \):

<table>
<thead>
<tr>
<th>channel 1</th>
<th>particle 1</th>
<th>particle 2</th>
<th>particle 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( K )</td>
<td>( N )</td>
<td>( N )</td>
</tr>
<tr>
<td>channel 2</td>
<td>( \pi )</td>
<td>( \Sigma )</td>
<td>( N )</td>
</tr>
<tr>
<td>channel 3</td>
<td>( \pi )</td>
<td>( N )</td>
<td>( \Sigma )</td>
</tr>
</tbody>
</table>
Many-channel operators:

\[ T_i \rightarrow T_i^{\alpha\beta} : \]

\[
T_1 = \begin{pmatrix}
T_{1^{NN}} & 0 & 0 \\
0 & T_{1^{\Sigma N}} & 0 \\
0 & 0 & T_{1^{\Sigma N}}
\end{pmatrix}, \quad T_2 = \begin{pmatrix}
T_{2^{KK}} & 0 & T_{2^{K\pi}} \\
0 & T_{2^{\pi N}} & 0 \\
T_{2^{\pi K}} & 0 & T_{2^{\pi\pi}}
\end{pmatrix}, \quad T_3 = \begin{pmatrix}
T_{3^{KK}} & T_{3^{K\pi}} & 0 \\
T_{3^{\pi K}} & T_{3^{\pi\pi}} & 0 \\
0 & 0 & T_{3^{\pi N}}
\end{pmatrix}
\]

\( T^{NN}, T^{\Sigma N} \) and \( T^{\pi N} \) are the usual \( T \)-matrixes, \( T^{KN-\pi\Sigma} \) is a 2-channel one:

\( T^{KK} : KN \rightarrow KN \quad T^{\pi K} : KN \rightarrow \pi\Sigma \)

\( T^{K\pi} : \pi\Sigma \rightarrow KN \quad T^{\pi\pi} : \pi\Sigma \rightarrow \pi\Sigma \)

Other operators: \( G_0 \rightarrow G_0^{\alpha\beta} = \delta_{\alpha\beta} G_0^\alpha, \quad U_{ij} \rightarrow U_{ij}^{\alpha\beta} \)

Separable T-matrices, depending on 2-isospins (isospin basis: the pair isospins are fixed, not individual projections, which correspond to a given particle composition; 2-body isospin is conserved)

\[
T_{i,l}^{\alpha\beta} = \left| g_{i,l}^{\alpha} \right> \tau_{i,l}^{\alpha\beta} \left< g_{i,l}^{\beta} \right| \quad \text{at } \alpha=\beta \text{ it is a usual } T\text{-matrix}
\]
Many-channel generalization of usual "separable" kernels and new unknown functions:

\[ Z_{ij,li,lj}^{\alpha\beta} = \delta_{\alpha\beta} \langle g_{i,l_i}^\alpha | G_0^\alpha | g_{j,l_j}^\beta \rangle, \quad X_{ij,li,lj}^{\alpha\beta} = \langle g_{i,l_i}^\alpha | G_0^\alpha U_{ij,li,lj}^{\alpha\beta} | g_{j,l_j}^\beta \rangle \]

Total 3-spin \( S = 0 \) \( \Rightarrow \) all 2-body interactions are in \( s = 0 \) states; \( l = 0 \);
total 3-isospin \( I = 1/2 \) (the lowest state):

\( X_{ij} \) (3 operators) \( \rightarrow X_{ij}^{\alpha\beta} \) (27 operators) \( \rightarrow X_{ij,li,lj}^{\alpha\beta} \) (51 unknown operators)

Search for a bound state or resonance - solving a homogeneous system of equations: 3 equal systems of 27 equations on \( X \)

Two identical nucleons in \( KNN \) channel: \underline{antisymmetrization}

Finally: system of 9 integral equations to be solved
Two-body interactions

\[ T^{NN}_{I}(k, k'; z) \] corresponds to

\[ V^{NN}_{I}(k, k') = -g^{NN}_{I}(k)g^{NN}_{I}(k') \quad \text{with} \quad g^{NN}_{I}(k) = \frac{1}{2\sqrt{\pi}} \sum_{i=1}^{6} \frac{c^{NN}_{i,I}}{k^2 + \left(\beta^{NN}_{i,I}\right)^2} \]

A separabilization of a Paris potential; \( I = 0 \) or \( 1 \) (for \( pp \) \( I = 1 \) only)

Taken from


Gives:

\[ E_{\text{deuteron}} = -2.2249 \text{ MeV}, \]
\[ a\left( ^3S_1 \right) = -5.422 \text{ fm}, \]
\[ a\left( ^1S_0 \right) = 17.534 \text{ fm}, \]
\( T_{I}^{\Sigma N}(k, k'; z) \) corresponds to

\[
V_{I}^{\Sigma N}(k, k') = \lambda_{I}^{\Sigma N} g_{I}^{\Sigma N}(k) g_{I}^{\Sigma N}(k')
\]

with

\[
g_{I}^{\Sigma N}(k) = \frac{1}{k^2 + \left( \beta_{I}^{\Sigma N} \right)^2}
\]

Parameters were fitted to reproduce:

\( I = 3/2: \)
1. \( a(I = 3/2) = 3.8 \text{ fm}, r_{\text{eff}}(I = 3/2) = 4.0 \text{ fm} \)

(from M.M. Nagels, T.A. Rijken, and J.J. de Swart, Phys. Rev. D20, 1633 (1979), Nijmegen potential)

2. Low-energy phase shifts of the most recent version of the same potential,

\( a(I = 3/2) = 4.15 \text{ fm}, r_{\text{eff}}(I = 3/2) = 2.4 \text{ fm} \)

\( I = 1/2: \)

\( a(I = 1/2) = -0.5 \text{ fm} \)

and some reasonable values of \( \lambda_{1/2} \) and \( r_{\text{eff}} \).
\( KN - \pi \Sigma \) two-channel interaction

\[ T_i^{\alpha\beta}(k^\alpha, k^\beta'; z) \] corresponds to

\[ V_i^{\alpha\beta}(k^\alpha, k^\beta') = g_i^{\alpha}(k^\alpha) \lambda_i^{\alpha\beta} g_i^{\beta}(k^\beta') \] with \[ g_i^{\alpha}(k) = \frac{1}{k^2 + (\beta_i^{\alpha})^2}, \]

\( \alpha, \beta = K \) (\( KN \) channel) or \( \pi \) (\( \pi \Sigma \) channel)

Parameters were fitted to reproduce:

- Energy of \( \Lambda(1405) \): \( E_\Lambda = 1406.5 - i \cdot 25 \text{ MeV} \),
  a resonance in \( \pi \Sigma \) and a quasi-bound state in \( KN \) channel \( (I = 0) \)

- \( \gamma = \frac{\sigma(K^- p \rightarrow \pi^+ \Sigma^-)}{\sigma(K^- p \rightarrow \pi^- \Sigma^+)} = 2.36 \), threshold branching ratio
  \textit{D.N. Tovee et. al., Nucl. Phys. B33, 493 (1971)}

- Cross-sections of \( K^- p \rightarrow K^- p \) and \( K^- p \rightarrow \pi^+ \Sigma^- \) reactions
• $K^- p$ scattering length:

$$a_{K^- p} = (-0.78 \pm 0.15 \pm 0.03) + i(0.49 \pm 0.25 \pm 0.12) \text{ fm}$$

from M. Iwasaki et al., Phys. Rev. Lett. 78, 3067 (1997);

Manifestation of $\Lambda(1405)$ resonance in $\pi \Sigma \rightarrow \pi \Sigma (I=0)$ cross-section
Due to uncertainties it is possible to choose different sets of parameters:

1. $a_{K^-p} = -0.78 + i \cdot 0.49$ fm
2. $a_{K^-p} = -0.78 + i \cdot 0.65$ fm
3. $a_{K^-p} = -0.70 + i \cdot 0.60$ fm

$T_{ij}^{\pi N}(k,k',z)=0$ because $\pi N$ interaction has a weak s-wave part
The results

Test three-body calculation (KNN channel only, Λ(1405) is a “real” bound state): a bound state in KNN with $E_{\text{bound}} = -43.7$ MeV was found

Full three-body calculations, 2 x 3 sets of parameters ($\Sigma N$ and KNN-$\pi\Sigma$)

<table>
<thead>
<tr>
<th>$a_{K^-p}$</th>
<th>$(a_{3/2}^{\Sigma N} = 3.8, \ r_{3/2}^{\Sigma N} = 4.0)$</th>
<th>$(a_{3/2}^{\Sigma N} = 4.15, \ r_{3/2}^{\Sigma N} = 2.4)$</th>
<th>no $\Sigma N$ interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.78 + i \cdot 0.49$</td>
<td>$-55.84 - i \cdot 49.11$</td>
<td>$-56.24 - i \cdot 50.12$</td>
<td>$-53.40 - i \cdot 49.21$</td>
</tr>
<tr>
<td>$-0.78 + i \cdot 0.65$</td>
<td>$-69.43 - i \cdot 46.83$</td>
<td>$-70.04 - i \cdot 47.93$</td>
<td>$-66.32 - i \cdot 47.47$</td>
</tr>
<tr>
<td>$-0.70 + i \cdot 0.60$</td>
<td>$-66.04 - i \cdot 54.71$</td>
<td>$-66.53 - i \cdot 55.77$</td>
<td>$-63.45 - i \cdot 54.59$</td>
</tr>
</tbody>
</table>

The obtained $E_{\text{res}} = -i \cdot \frac{\Gamma}{2}$ for 9 sets of the input parameters,

the $E_{\text{res}}$ is measured from the $K^-pp$ threshold
Conclusions

**Full three-body 2-channel** calculation:

A resonance in $I = 1/2$, $J^\pi = 0^-$ $KNN-\pi\Sigma N$ was found

at $E_{res} \sim 60$ MeV with $\Gamma/2 \sim 50$ MeV

- Inclusion of the second channel is very important
- The position of the resonance strongly depends on parameters of the $KN-\pi\Sigma$ interaction
- Inclusion of $\Sigma N$ is important, but the results rather weakly depends on parameters of the interaction