Kaon Condensation in Neutron Star using Modified Quark-Meson Coupling Model

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Outline

• Introduction (Strangeness in neutron star)
• Motivation of this work (model dependency)
• Models
  - Quantum Hadrodynamics (QHD) model
  - Quark-Meson Coupling (QMC) model
  - Modified Quark-Meson Coupling (MQMC) model
• Neutron star matter
  - kaon condensation in neutron star matter with hyperons
• Results and summary
Does strangeness matter in neutron star?

Mass of nucleon star (n, p, e) \( \sim (2.0 - 2.9) M_\odot \)
(no strangeness)

1. Hadronic model (QHD)

Knorren, Parkash, and Ellis, PRC52, 3470 (1995)

<table>
<thead>
<tr>
<th>np</th>
<th>Without kaons</th>
<th>( \frac{M_{\text{max}}}{M_\odot} )</th>
<th>( u_{\text{cent}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H300</td>
<td></td>
<td>2.529</td>
<td>5.13</td>
</tr>
<tr>
<td>H200</td>
<td></td>
<td>2.508</td>
<td>5.32</td>
</tr>
<tr>
<td>GM1</td>
<td></td>
<td>2.346</td>
<td>5.70</td>
</tr>
<tr>
<td>GM2</td>
<td></td>
<td>2.064</td>
<td>6.58</td>
</tr>
<tr>
<td>GM3</td>
<td></td>
<td>2.005</td>
<td>7.14</td>
</tr>
<tr>
<td>B91</td>
<td></td>
<td>2.097</td>
<td>5.80</td>
</tr>
<tr>
<td>HS81</td>
<td></td>
<td>2.954</td>
<td>3.85</td>
</tr>
</tbody>
</table>
Does strangeness matter in neutron star?

2. Quark models

2-1. Quark-Meson Coupling Model


<table>
<thead>
<tr>
<th>Type</th>
<th>Hadron model</th>
<th>$M_{\text{max}}(M_{\odot})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>np</td>
<td>QMC</td>
<td>2.20</td>
</tr>
</tbody>
</table>

2-2. Modified Quark-Meson Coupling Model

Pal, Hanuske, Zakout, Stoecker, & Greiner, PRC 60, 15802 (1999)

<table>
<thead>
<tr>
<th>$M_{\text{max}}/M_{\odot}$</th>
<th>QMC</th>
<th>$R_{M_{\text{max}}}$ (km)</th>
<th>$n_e$ (fm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>np</td>
<td>1.988</td>
<td>10.632</td>
<td>1.102</td>
</tr>
</tbody>
</table>
Need something more than just nucleons.

Theoretical masses are in this range without strangeness.


Vertical lines are at $1.35 \pm 0.04 M_\odot$

Mass of neutron stars

neutron star-white dwarf binaries

neutron star binaries
Does strangeness matter in neutron star?

Mass of \textit{strange} stars (with \textit{hyperons and/or kaons}) \((1.5 - 2.0) M_\odot\)

2-1. **QHD with H**
Knorren, Parkash, and Ellis, PRC52, 3470 (1995)

\[
\begin{array}{|c|c|c|}
\hline
npH & GM1 & 1.776 \\
& GM2 & 1.655 \\
& GM3 & 1.544 \\
& B91 & 1.463 \\
& HS81 & 1.38 \\
\hline
\end{array}
\]

2-2. **QMC with H and/or K**
Menezes, Panda, and Providencia, PRC72, 35802 (2005)

\[
\begin{array}{|c|c|c|}
\hline
np + hyperons & QMC & 1.98 \\
np + hyperons + kaon & QMC & 1.94 \\
\hline
\end{array}
\]

2-3. **MQMC with H**
Pal, Hanauske, Kaout, Stoecker, & Greiner, PRC60, 15802 (1999)

\[
\begin{array}{|c|c|c|c|}
\hline
npH (model I) & 1.478 & 11.242 & 0.965 \\
npH (model II) & 1.539 & 10.823 & 1.096 \\
\hline
\end{array}
\]

(No kaons)
Motivation 1

- Different results from different models.

<table>
<thead>
<tr>
<th>Models</th>
<th>$M/M_\odot$</th>
<th>$\rho_c/\rho_0$ for Kaon Condensation</th>
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<td>QHD (GM1)</td>
<td>1.649</td>
<td>2.23</td>
<td>Banik et al</td>
</tr>
<tr>
<td>QMC</td>
<td>1.940</td>
<td>4.0</td>
<td>Menezes et al</td>
</tr>
</tbody>
</table>

Banik and Bandyopadhyay, PRC64, 55805 (2001)
Glendenning and Moszkowski, PRL67, 2414 (1991)

- Model dependency: Different model calculations use different Lagrangians, model parameters, saturation properties.

- By using QHD and MQMC models with essentially the same Lagrangian, parameters and saturation properties, we can check the model dependency.
By changing the **kaon optical potential**,

- **composition** of neutron star matter
- **onset density** of kaon condensation in neutron star matter
- **maximum mass** of a neutron star
1) Quantum Hadrodynamics (QHD)  
hadron degrees of freedom  

2) Quark-meson coupling (QMC) model  
quark degrees of freedom  

3) Modified Quark-meson coupling (MQMC) model  
QMC + density-dependent bag constant  
- **The extended MQMC model for baryon octet (with hyperons and $\sigma^*$ and $\phi$ mesons):**

  Pal, Hanauske, Zakout, Stoecker, & Greiner, PRC 60, 15802 (1999)

- **Kaon Lagrangian:**

  Glendenning and Schaffner-Bielich, PRC60, 25803 (1999)

  \[
  \mathcal{L} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_K + \mathcal{L}_l
  \]

  \[
  \mathcal{L}_B = \sum_B \bar{\psi}_B \left[ i \gamma \cdot \partial - m^*_B(\sigma, \sigma^*) - \gamma^0 \left( g_{\omega B} \omega_0 + g_{\phi B} \phi_0 + \frac{1}{2} g_{\rho B} \tau z \rho_0 \right) \right] \psi_B
  \]

  \[
  \mathcal{L}_M = \frac{1}{2} \left( -m^2_\sigma \sigma^2 - m^2_\sigma^* \sigma'^2 + m^2_\omega \omega^2 + m^2_\phi \phi^2 + m^2_\rho r^2_0 \right) - U_{\text{QHD}}(\sigma)
  \]

  \[
  \mathcal{L}_K = D^*_\mu K^* D^\mu K - m^2_K K^* K
  \]

  \[
  \mathcal{L}_l = \sum_l \bar{\psi}_l \left( i \gamma \cdot \partial - m_l \right) \psi_l
  \]

  \[
  U_{\text{QHD}}(\sigma) = \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4
  \]

  \[
  D_\mu = \partial_\mu + ig_{\omega K} \omega_\mu - ig_{\phi K} \phi_\mu + i \frac{1}{2} g_\rho \tau \cdot \rho_\mu
  \]

  \[
  m^*_B(\text{QHD}) = m_B - g_{\sigma B} - \sigma g_{\sigma B} \sigma^*
  \]

  \[
  m^*_B(\text{MQMC}) = \sqrt{E^2_B - \sum_q \left( \frac{x_q}{R} \right)^2}
  \]

  \[
  m^*_K = m_K - g_{\sigma K} \sigma - g_{\sigma K} \sigma^*
  \]
QHD vs (M)QMC : Treatment of baryons

Quantum Hadrodynamics (QHD)

Hadron degree of freedom:

\[ \sigma, \omega \]

Quark-meson coupling model (QMC) &

Modified Quark-meson coupling model (MQMC)

Quark degree of freedom:

\[ \sigma, \omega \]
Modified QMC (MQMC) model

Quark-meson coupling (QMC) model

(MIT bag model + meson interactions)


MQMC model

Density dependent bag constant $B_N(\sigma)$

$$B_B(\sigma, \sigma^*) = B_{B0} \exp \left\{ -4g^B_{\sigma} \left( \sum_{q=u,d} n_q \sigma + (3 - \sum_{q=u,d} n_q) \sqrt{2} \sigma^* \right) / m_B \right\}$$

Jin and Jennings, PRC 54, 1427 (1996)
Coupling constants

**Hyperon**

- **Quark counting**:
  
  \[ g_{MH} = \frac{\sum_{q=u,d} n_i^q}{3} g_{MN} \]

- **SU(6) Symmetry**:
  
  \[ g_{\sigma^*H} = \sqrt{2} g_{\sigma H}, \quad g_{\phi H} = \sqrt{2} g_{\omega H} \]

  \[ g_{\sigma_Y} = \frac{2 g_{\sigma_N}}{3}, \quad g_{\omega_Y} = \frac{2 g_{\omega_N}}{3}, \quad g_{\rho_{\Sigma,\Xi}} = g_{\rho_N} \]

  \[ g_{\sigma^*_{\Lambda,\Sigma}} = \sqrt{2} g_{\sigma_Y}/3, \quad g_{\sigma^*_{\Xi}} = \frac{2 \sqrt{2}}{3} g_{\sigma_N}/3 \]

  \[ g_{\phi_{\Lambda,\Sigma}} = \sqrt{2} g_{\omega_N}/3, \quad g_{\phi_{\Xi}} = \frac{2 \sqrt{2}}{3} g_{\omega_N}/3 \]

**Kaon**

- \[ g_{\omega_K} = \frac{g_{\omega_N}}{3}, \quad g_{\rho_K} = g_{\rho_N} \text{ (quark counting)} \]

- \[ g_{\sigma^*_K} : \text{decay of } f_0(980) \text{ (WA76 collaboration)} \]

- \[ g_{\phi_K} = g_{\pi\pi\rho}/\sqrt{2} \text{ (quark counting + SU(6))} \]

- \[ g_{\sigma_K} : \quad U_{K^-}(\rho_0) = -g_{\sigma K} \sigma(\rho_0) - g_{\omega K} \omega(\rho_0) \]

\[ U_{K^-}(\rho_0) = -120, -140, -160 \text{ MeV} \]
16 Relations for 16 unknowns for neutron star matter

Variables : 16 in total

9 hadron densities (8 octet baryons + K⁻)
5 meson fields (σ, ω, ρ, σ*, φ)
2 lepton densities (e, m)

Conditions : 16 in total

Baryon number conservation (1)
Charge neutrality (1)
Chemical equilibrium (9)
Eqs. of motion for meson fields (5)

- 16 equations are solved self-consistently.
- Highly nonlinear for MQMC model.
Composition for different $K^-$ optical potential

$$U_{K^-}(\rho_0) = -120 \text{ MeV}$$

$\rho_c = 9.8 \rho_0$

$\rho_c = 5.9 \rho_0$
Composition for different $K^-$ optical potential

\[ U_{K^-}(\rho_0) = -140 \text{ MeV} \]

**QHD**

\[ \rho_c = 4.3 \rho_0 \]

**MQMC**

\[ \rho_c = 3.8 \rho_0 \]
Composition for different $K^-$ optical potential

$$U_{K^-}(\rho_0) = -160 \text{ MeV}$$

**QHD**

$$\rho_c = 3.3\rho_0$$

**MQMC**

$$\rho_c = 3.0\rho_0$$
### Mass and $\rho_{\text{crit}}$ from QHD and MQMC

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<th>$\rho_{\text{crit}}/\rho_0$ for Kaon Condensation</th>
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Summary

- By using similar Lagrangians and parameters and the same saturation properties, model dependency (QHD vs MQMC) is studied.

- Two models give us different compositions, mass and the kaon condensation onset density.

- Quark model results in an earlier onset of kaon condensation.

- Onset of kaon condensation is sensitive to the kaon optical potential.

- A deeper kaon-nucleus optical potential causes softening of the equation of state.
Calculation of kaon optical potential and $g_{\sigma K}$ from a quark model

1. Calculate the effective mass of kaon in matter in two ways.

   a) kaon as a point particle :

      Kaon Lagrangian :

      \[ L_K = D_\mu^* K^* D_\mu K - m_K^* K^* K \]

      \[ D_\mu = \partial_\mu + ig_{\omega K} \omega_\mu \]

      \[ m_K^* = m_K - g_{\sigma K} \sigma \]  

      (Glendenning & Schaffner, PRC60, 25803 (1999). )

      Real part of $K$ optical potential at saturation density

      \[ U_{K^-} (\rho_0) = -g_{\sigma K} \sigma (\rho_0) - g_{\omega K} \omega (\rho_0) \]

   b) kaon as an meson (MIT) bag :

      \[ m_K^* = \sqrt{E_K^2 - \sum_{i=q,\bar{q}} \left( \frac{x_i}{R} \right)^2}, \quad E_K = \sum_{i=q,\bar{q}} \frac{\Omega_i}{R} - \frac{Z_K}{R} + \frac{4}{3} \pi R^3 B_K \]

2. Set the effective mass from (a) equal to the mass from (b).
The effective mass of kaon and $g_\sigma K$

$g_\sigma K \approx 2$ and $U_K = -100$ MeV

$m_K^* (\text{MeV})$

$\rho / \rho_0$

$m_s = 150 \text{ MeV}$

$R_{K^0} = 0.6 \text{ fm}$