and $\Xi$ Hypernuclei in a chiral SU(3) RMF model

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KT-AO, nucl/th:0607046; HM-AO, in preparation.
Chiral symmetric RMF

- Relativistic Mean Field model
  - Hadronic model which is very powerful tools to investigate nuclear matter and finite nucleus.
  - Symmetries: works as constrants to $B-M$ coupling and meson self-interaction.
- Problems in chiral symmetric RMF model

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Problems in chiral symmetric RMF model

1. KT-AO, nucl/th:0607046.

Energy Density at $\rho_B=(0-5)\rho_0$

- $\phi^4$
- $\text{Bog. } (\sigma^2 \omega^2)$
- $\text{BL } (-\sigma^4 \log \sigma^2)$
- $\text{SCL } (-\log \sigma^2)$
- $\text{Fermi integral}$
- $\text{TM1}$

$\Sigma$ and $\Xi$ Hypernuclei in a chiral SU(3) RMF model – p. 2
Chiral symmetric RMF

- Relativistic Mean Field model
  - Hypernuclei
  - Symmetries: works as constants to B-M coupling and meson self-interaction.

Problems in chiral symmetric RMF model

Problem:

Sudden chiral restoration below $\rho_0$ [2]

Too stiff EOS [3, 4]


$$ U = a \log(\det M_{\text{SU(2)}}) + b \text{tr} \left( M_{\text{SU(2)}} \right) + c $$

$$ = 2a \log(1 + f^2) + 1/2 f^2 + 1/2 m^2 + 1/2 m^2 $$

Sudden chiral restoration: suppressed by log barrier

Qualitatively agreement with TM1 model [6]

$E/V$ vs $\sigma$ for different models.
Chiral symmetric RMF

- Relativistic Mean Field model
- Hadronic model which is very powerful tools to investigate nuclear matter and finite nucleus.
- Symmetries: works as constraints to $B$-$M$ coupling and meson self-interaction.

Problems in chiral symmetric RMF model

1. KT-AO, nucl/th:0607046.

We use scalar meson self-interaction derived from Strong Coupling Lattice QCD [5]

$$U_\sigma = -a \log(\det M_{SU(2)}M_{SU(2)}^\dagger) + b \text{tr}(M_{SU(2)}M_{SU(2)}^\dagger) + c_\sigma \sigma$$

$$= -2a \left\{ \log \left( 1 + \frac{\sigma}{f_\pi} \right) + \frac{\sigma}{f_\pi} - \frac{1}{2} \left( \frac{\sigma}{f_\pi} \right)^2 \right\} + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\pi^2 \pi^2$$

[1] KT-AO
[3] Ogawa

$\Sigma$ and $\Xi$ Hypernuclei in a chiral SU(3) RMF model – p. 2
Chiral symmetric RMF

- Relativistic Mean Field model
  - Hinders sudden chiral restoration
  - Symmetries: work as constraints to $B$-$M$ coupling and meson self-interaction.

Problems in chiral symmetric RMF model


Sudden chiral restoration: suppressed by log barrier

Energy Density at $\rho_B=(0-5)\rho_0$

- $\phi^4$
- Bog. ($\sigma^2\omega^2$)
- BL ($-\sigma^4\log\sigma^2$)
- SCL ($-\log\sigma$)
- Fermi integral
- TM1

$\Sigma$ and $\Xi$ Hypernuclei in a chiral SU(3) RMF model – p. 2
Chiral symmetric RMF

- Relativistic Mean Field model
  - Hadronic model which is very powerful tools to investigate nuclear matter and finite nucleus.
  - Symmetries: works as constraints to $B$-$M$ coupling and meson self-interaction.

Problems in chiral symmetric RMF model

1. KT-AO, nucl/th:0607046.

Sudden chiral restoration: suppressed by log barrier

Energy Density at $\rho_B=(0-5)\rho_0$

- Qualitatively agreement with TM1 model

Binding energy per nucleon (MeV)

Different models: exp. SCL TM1 and 2

- Heavier side
- Lighter side

Hypernuclei in a chiral SU(3) RMF model – p. 2
Chiral symmetric RMF

- Relativistic Mean Field model
  - Has powerful tools to investigate nuclear matter and finite nucleus.
  - Symmetries: constrains to $B$-meson self-interaction.

Problems in chiral symmetric RMF model:

1. KT-AO, nucl/th:0607046.

Sudden chiral restoration: suppressed by log barrier

Qualitatively agreement with TM1 model.

- Binding energy per nucleon (MeV):
  - Lighter side: C, O, Ca, Ni, Zr, Sn
  - Heavier side: Pb

- Hypernuclei in a chiral SU(3) RMF model – p. 2
Simple extension for hyperonic system:

\[ U_{\sigma,\zeta} = -a \log(\det M_{\text{SU}(3)} M_{\text{SU}(3)}^\dagger) + b \text{tr}(M_{\text{SU}(3)} M_{\text{SU}(3)}^\dagger) \]

\[ + c_{\sigma} \sigma + c_{\zeta} \zeta + d \det(M_{\text{SU}(3)} + M_{\text{SU}(3)}^\dagger) \]

\[ = -a \left( 2f_{\text{SCL}} \left( \frac{\sigma}{f_{\pi}} \right) + f_{\text{SCL}} \left( \frac{\zeta}{f_{\zeta}} \right) \right) \]

\[ + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_{\pi}^2 \pi^2 + \frac{1}{2} m_{\zeta}^2 \zeta^2 + \frac{1}{2} m_K^2 K^2 \]

\[ + \xi_{\sigma,\zeta} \sigma \zeta \]

\[ f_{\text{SCL}}(t) = \log(1 + t) + t - \frac{1}{2} t^2 \]

\[ a = \frac{f_{\pi}^2}{4} \left( m_\sigma^2 - m_{\pi}^2 \right) \]
Simple extension for hyperonic system:

\[
U \equiv a \log(\det(M_{SU(3)} M_y)) + b \text{tr}(M_{SU(3)} M_y) + c + d \det(M_{SU(3)} + M_y)
\]

\[
E_{0/V} = 0.4 + 0.2 \sigma_f SCL(\sigma_f)
\]

Energy surface at \( \rho_B = 0 \)

EOS should be softened by \( \sigma - \zeta \) mixing (\( k \sim 220 \text{MeV} \)).
Simple extention for hyperonic system:

\[ U = a \log(\det M_{SU(3)} M_y) + b \text{tr}(M_{SU(3)} M_y) + c + d \det(M_{SU(3)} + M_y) = a \frac{2}{f_{SCL}}(f_{SCL}) + f_{SCL}(t) = \log(1+t) + t \frac{1}{2} t^2 \]

Energy surface at \( r_B = 0 \):

\[ E_0/V = -0.4 \]

Hypernuclei in a chiral SU(3) RMF model – p. 3

Sepalation energy of \( \Lambda \): reproduced nicely.
\( \Lambda \) in Chiral SU(3) RMF

- Simple extension for hyperonic system:

\[
U_{\sigma,\pi} = a \log(\det(M_{SU(3)} + M_y)) + b \text{tr}(M_{SU(3)} M_y) + c + d \det(M_{SU(3)} + M_y)
\]

\[
= a f_{SCL}(f) + f_{SCL}(t) = \log(1 + t) + t \frac{1}{2} \frac{1}{2} = \frac{1}{2} \frac{1}{2} m^2 \frac{1}{2} + \frac{1}{2} \frac{1}{2} m^2 K K
\]

Energy surface at \( r_B = 0 \):

\[
E_0/V = -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
\]

\[
s_p/f_z = -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
\]

\[
NAGARA
\]

\( \Delta B_{\Lambda\Lambda} \) of \( ^6\Lambda\Lambda\text{He} \)

\( \Delta B_{\Lambda\Lambda} \) vs \( g_{\zeta\Lambda}/g_{\zeta\Lambda} \)

- All of parameters about \( \Lambda \): decided by fitting single and double \( \Lambda \) hypernuclei.
hyper\textquotedblleft atom\textquotedblright

- $\Sigma$: repulsive in nucleus \[7\]
- Atomic shift of $\Sigma^-$ suggests $\Sigma$-N interaction should be attractive around nuclear surface\[8, 9\].

Parameter search: to explain $\Sigma^-$ atomic shifts.

- Vector part (repulsion) $g_{\omega\Sigma}$: two cases are tested.
  - From SU(3) coupling relation and OZI rule
    \[ g_{\omega\Sigma} = \frac{g_{\omega N} + g_{\rho N}}{2} \approx \frac{1}{3} g_{\omega N} \rightarrow \text{Weak repulsion} \]
  - $g_{\omega\Sigma} \approx \frac{2}{3} g_{\omega N}$ (Mares et al.) \rightarrow \text{Strong repulsion}

- Scalar part (attraction) ($g_{\sigma\Sigma}$ and $g_{\zeta\Sigma}$): chosen so as to reproduce atomic shifts of N=Z core nuclei (O, Si, S).

- $g_{\rho\Sigma}$: adjusted to get correct atomic shift in Pb.
Results(1)

Parameter search: to explain $\Sigma^-$ atomic shifts.

Vector part (repulsion): two cases are tested.

From SU(3) coupling relation and OZI rule,

\[ g = \frac{g_N}{2} + \frac{g_N}{2} \]

Weak repulsion

\[ g = \frac{g_N}{3} \] (Mares et al.)

Strong repulsion

Scalar part (attraction) \((g_\text{sr}\text{ and } g_\text{wr})\): chosen so as to reproduce atomic shifts of N=Z core nuclei (O, Si, S).

\[ g \] adjusted to get correct atomic shift in Pb.

\begin{table}[h]
\begin{tabular}{|c|c|c|c|}
\hline
Z & \text{SR} & \text{WR} & \text{Exp.} \\
\hline
4 & 3 & 5 & 10 \\
5 & 4 & 6 & 10 \\
10 & 9 & 10 & 9 \\
\hline
\end{tabular}
\end{table}

\[ \chi^2 / \text{dof} \sim 1.3 \]
Conversion width: calculated perturbatively as expectation value of $\text{Im} V_{\text{opt}} = t\rho_p$.
Results(2)

- Conversion width: calculated perturbatively as expectation value of $\text{Im}V_{\text{opt}} = t\rho_p$.

- Depth of $\text{Im}V_{\text{opt}}$: $-15 \sim -20 \text{ MeV}$. 

\[ \Sigma \text{ and } \Xi \text{ Hypernuclei in a chiral SU(3) RMF model} – p. 6 \]
Conversion width: calculated perturbatively as expectation value of $\text{Im}V_{\text{opt}} = t\rho_p$.

Depth of $\text{Im}V_{\text{opt}}$: $-15 \sim -20$ MeV.
We succeeded in reproducing the atomic shifts of $\Sigma^-$ with Chiral SU(3) symmetric RMF model.

- Attractive pocket: a few MeV.
- Depth of Im$V_{\text{opt}}$: $-15 \sim 20$ MeV

Simple SU(3) coupling relation: not works well when we calculate $N > Z$ nuclide.

- It may not be good constraint for $\Sigma^-$. 

We construct chiral SU(3) RMF model which can explain nuclear matter, finite nuclei, $\Lambda$ hypernuclei and $\Sigma^-$ atomic data.
Future works

- Check a consistency between quasi-free spectrum.
  - We have investigated $\Sigma^-$ quasi-free spectrum with DWIA+Local Optimized Fermi Average (Maekawa’s talk at parallel C2). We would like to check whether $\Sigma^-$ repulsion should be strong or relative weak.

- Application to $\Xi$ hypernucleus
  - Depth of SE potential: $U_{\Xi} \sim -15\text{MeV}$.
  - Constraints: flavor SU(3) symmetry for $\Xi$-meson coupling
  - What nuclide does $\Xi$ hyperon has bound state in?

- For the description to nuclear star or SN.
That’s all.

Thank you for listening!
Chiral SU(3) model

- Application to systems including hyperon
  - Needed to include $\zeta$ and $\phi_0$ meson.
  - Problems: how to deal with scalar meson self-interaction?
  - We can calculate simple extension from our chiral SU(2) one.

\[
U_{\sigma\zeta} = -\frac{a}{2} \det \left( M^\dagger M \right) + btr \left( M^\dagger M \right) + C_\sigma \sigma + C_{\zeta\zeta} + d \left( \det M + \det M^\dagger \right)
\]

\[
= -a \left[ \left\{ \log \left( 1 + \frac{\sigma}{f_\pi} \right) + \frac{\sigma}{f_\pi} - \frac{\sigma^2}{2 f_\pi} \right\} + \frac{1}{2} \left\{ \log \left( 1 + \frac{\sigma}{f_\zeta} \right) + \frac{\sigma}{f_\zeta} - \frac{\zeta^2}{2 f_\zeta} \right\} \right] \\
+ \frac{m_\sigma}{2} \sigma^2 + \frac{m_\zeta^2}{2} \zeta^2 + \xi_{\sigma\zeta} \sigma \zeta + \frac{m_\pi}{2} \pi^2 + \frac{m_K^2}{2} K^2
\]

\[
\mathcal{L}^{\text{SCL-SU(3)}} = \mathcal{L}^{\text{SU(3) baryon}} + \mathcal{L}^{\text{SU(3) meson}} + V_{\sigma\zeta} + \xi_{\sigma\zeta} \sigma \zeta + D_\omega \omega^4
\]

- Parameters: $g_{\omega N}$, $g_{\rho N}$, $g_{\omega \Lambda}$, $g_{\phi \Lambda}$, $g_{\sigma \Lambda}$, $g_{\zeta \Lambda}$, $D_\omega$ and $m_\sigma$ → decided so as to reproduce symmetric nuclear matter, normal nuclei, single and double $\Lambda$ hypernuclei.
## Atomic Shifts

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>WR</th>
<th>SR</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>318</td>
<td>289</td>
<td>320 (220)</td>
</tr>
<tr>
<td>Si</td>
<td>148</td>
<td>159</td>
<td>159 (36)</td>
</tr>
<tr>
<td>S</td>
<td>493</td>
<td>447</td>
<td>360 (220)</td>
</tr>
<tr>
<td>Pb</td>
<td>421</td>
<td>415</td>
<td>422 (56)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>WR</th>
<th>SR</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg</td>
<td>40</td>
<td>45</td>
<td>25 (40)</td>
</tr>
<tr>
<td>Al</td>
<td>78</td>
<td>86</td>
<td>68 (28)</td>
</tr>
<tr>
<td>W</td>
<td>104</td>
<td>112</td>
<td>214 (60)</td>
</tr>
</tbody>
</table>
Scalar part

Total $\chi^2$ dependency

$g_{\Sigma} \times g_{\Xi}$

$g_{\sigma\Sigma}$

$\Sigma$ and $\Xi$ Hypernuclei in a chiral SU(3) RMF model – p. 12


