Self energies of the pion and the $\Delta$ isobar from the $^3$He(e,e$'$\pi$^+$)$^3$H reaction


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Abstract

In a kinematically complete experiment at the Mainz microtron MAMI, pion angular distributions of the $^3$He(e,e$'$\pi$^+$)$^3$H reaction have been measured in the excitation region of the $\Delta$ resonance to determine the longitudinal ($L$), transverse ($T$), and the $LT$ interference part of the differential cross section. The data are described only after introducing self-energy modifications of the pion and $\Delta$-isobar propagators. Using Chiral Perturbation Theory (ChPT) to extrapolate the pion self energy as inferred from the measurement on the mass shell, we deduce a reduction of the $\pi^+$ mass of $\Delta m_{\pi^+} = (-1.7^{+1.7}_{-2.1})$ MeV/c$^2$ in the

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neutron-rich nuclear medium at a density of \( \rho = (0.057^{+0.085}_{-0.057}) \) fm\(^{-3} \). Our data are consistent with the \( \Delta \) self energy determined from measurements of \( \pi^0 \) photoproduction from \(^4\)He and heavier nuclei. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

A basic question in hadronic physics concerns the properties of constituents as they are embedded in a nuclear medium. Such medium effects are commonly treated in terms of self energies from which effective masses and decay widths are deduced. Electroproduction of charged pions from \(^3\)He represents a viable testing ground to study the influence of the nuclear medium on the production and propagation of mesons and nucleon resonances such as the pion and the \( \Delta \) resonance. As a simple composite nucleus, \(^3\)He is amenable to precise microscopic calculations of the wave function and other ground state properties [1] and offers the great advantage that effects of final state interaction are expected to be much smaller than in heavier nuclei. Moreover, the mass-three nucleus may already be considered as a medium. In this Letter, we present the results of an experiment which allows the determination of the self energies of the pion and the \( \Delta \) isobar from the analysis of the longitudinal and transverse cross section components, respectively. These self-energy terms are the subject of theoretical descriptions in the framework of the \( \Delta \)-hole model [2] and Chiral Perturbation Theory (ChPT) [3].

2. Measurements

To this end, we have measured the \(^3\)He(\(e,e'\pi^+\))\(^3\)H reaction in a kinematically complete experiment at the high-resolution three-spectrometer facility [4] of the A1 collaboration at the 855 MeV Mainz microtron (MAMI). The specific experimental arrangements of the present experiment, including that of the cryogenic gas target and the data acquisition and analyses methods are described in detail in [5]. The very high missing mass resolution of \( \delta M \approx 700 \) keV/c\(^2\) (FWHM) is quite adequate [5–8] to clearly isolate the coherent channel (\(^3\)H\(\pi^+\)) from the three- and four-body final states (nd\(\pi^+\)) and (nn\(\pi^+\)).

The three-fold differential pion electroproduction cross section with unpolarized electron beam and target can be written as [9]

\[
\frac{d^3\sigma}{d\Omega_e' \cos \theta_e' d\Omega_{\pi}} = \Gamma \frac{d\sigma}{d\Omega_{\pi}}(W, Q^2, \theta_{\pi}, \phi_{\pi}, \epsilon),
\]

with

\[
\frac{d\sigma}{d\Omega_{\pi}} = \frac{d\sigma_T}{d\Omega_{\pi}} + \epsilon \frac{d\sigma_L}{d\Omega_{\pi}} + \epsilon \cos 2\phi \frac{d\sigma_{LT}}{d\Omega_{\pi}} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{d\Omega_{\pi}}
\]

Here the quantities \( \epsilon \) and \( \Gamma \) denote the polarization and flux of the virtual photon. The indices \( T, L, LT, \) and \( TT \) refer to the transverse and longitudinal components and their interferences, respectively. The explicit dependence of \( d\sigma/d\Omega_{\pi} \) on the azimuthal pion angle \( \phi_{\pi} \) and the polarization \( \epsilon \) is used for a separation of the response functions.

The measurements were carried out at two four-momentum transfers \( Q^2 = 0.045 \) and \( 0.100 \) (GeV/c\(^2\))^2, referred to as kinematics 1 and 2, respectively. The energy transfer in the laboratory frame has been chosen at \( \omega = 400 \) and 394 MeV, respectively, i.e., in the \( \Delta \) resonance region. At each \( Q^2 \), three measurements in parallel kinematics with various values of \( \epsilon \) were made to determine the \( L \) and \( T \) cross sections (Rosenbluth separation). Parallel kinematics implies that the pion is detected in the direction of the three momentum of the virtual photon. We have also measured the in-plane pion angular distribution (i.e., \( \phi_{\pi} = 0^\circ \) or \( 180^\circ \), respectively) for the second kinematics at \( \epsilon = 0.74 \) to determine the \( LT \) term. Parts of the experimental results together with model interpretations have already been presented elsewhere [5,8]. In this Letter, we offer a combined analysis of the entire data set of the experiments in the two kinematics and draw definitive con-
clinations about medium effects, which are especially well understood for the pion.

3. Results and discussion

The results of the Rosenbluth separation are shown in Fig. 1 where the cross sections are displayed as a function of the virtual photon polarization. The longitudinal cross section is identified as the slope, while the transverse one is given by the intercept with the axis at $\epsilon = 0$. Also shown in Fig. 1 are the fit results for the $L$ and $T$ components with statistical errors. The systematic errors amount to 10% (8%) for kinematics 1 (2), respectively. The theoretical calculations are based on the most recent elementary pion production amplitude in the framework of the so-called Unitary Isobar Model [9,10]. In Plane-Wave Impulse Approximation (PWIA), the amplitude includes the Born terms as well as $\Delta$- and higher resonance terms. For the mass-three nuclei, realistic three-body Faddeev wave functions are employed. In the Distorted-Wave (DWIA) calculations, the final state interaction due to pion rescattering is included [11]. As is seen in Fig. 1, the DWIA calculations underestimate the longitudinal component and overestimate the transverse component, each by about a factor of two. Since the longitudinal component is dominated by the pion-pole term and a large part of the transverse part arises from the $\Delta$ resonance excitation, both the pion and the $\Delta$ propagators have to be modified (see also [12]). In parallel kinematics the pion-pole term only contributes to the longitudinal part of the cross section, while the $\Delta$ excitation is almost purely transverse. Therefore, the pion-pole and the $\Delta$ contribution essentially decouple in the longitudinal and transverse channel and can be studied separately. We next discuss the estimate of these terms.

3.1. Modification of the pion

The inadequacy of the DWIA to account for the longitudinal response (cf. Fig. 1) is remedied by replacing the free pion propagator in the t-channel pion-pole term of the elementary amplitude, $[\omega_\pi^2 - \vec{q}_\pi^2 - m_\pi^2]^{-1}$, by a modified one, $[\omega_\pi^2 - \vec{q}_\pi^2 - m_\pi^2 - \Sigma_\pi(\omega_\pi, \vec{q}_\pi)]^{-1}$, where $\Sigma_\pi(\omega_\pi, \vec{q}_\pi)$ denotes the pion self energy in the nuclear medium [13]. For the two values of $Q^2$, the energy $\omega_\pi$ and the momentum $\vec{q}_\pi$ of the virtual pion are fixed as $\omega_\pi = 1.7$ (4.1) MeV and $|\vec{q}_\pi| = 80.9$ (141.2) MeV/c, such that two experimental numbers for $\Sigma_\pi$ can be determined from a fit to the respective longitudinal cross sections. The best-fit values result in $\Sigma_\pi = -(0.22 \pm 0.11) m_\pi^2$ for kinematics 1 and $\Sigma_\pi = -(0.44 \pm 0.10) m_\pi^2$ for kinematics 2. Close to the static limit, i.e., for $\omega_\pi \approx 0$, appropriate for the kinematical conditions of the present experiment, the pion self energy can be written as

$$\Sigma_\pi(0, \vec{q}_\pi) = -\frac{\sigma_N}{f_\pi^2} (\rho_p + \rho_n) - \vec{q}_\pi^2 \chi(0, \vec{q}_\pi),$$  

(2)

where $\rho_p$ and $\rho_n$ denote the proton and neutron densities, $\sigma_N = 45$ MeV the $\pi N$ sigma term [14], $f_\pi = 92.4$ MeV the pion decay constant, and $\chi(0, \vec{q}_\pi)$ the p-wave pionic susceptibility. Since the virtual $\pi^+$ propagates in a triton-like medium, we have $\rho_n = 2\rho_p$. In
infinite nuclear matter with Fermi momentum $p_F$, the p-wave pionic susceptibility $\chi(0, \vec{q}_\pi)$ can be approximated by a constant for $|\vec{q}_\pi| \lesssim p_F$, and we will assume that this is also the case here, although a local density approximation for such a small nucleus may be questionable. With the two values for $\Sigma_\pi$ given above, we immediately obtain $\chi = 0.31 \pm 0.22$. On the other hand, a standard calculation with particle-hole (ph) and $\Delta$-hole ($\Delta h$) susceptibilities (see, e.g., [13]) for infinite isospin-asymmetric nuclear matter results already at small densities in much higher values for $\chi$. For example, with $\rho_p + \rho_n = \frac{1}{3} \rho_0$ ($\rho_0 = 0.17$ fm$^{-3}$ being the saturation density), $\rho_0 = 2 \rho_p$ and the Migdal parameters $g'_{NN} = 0.8$ and $g'_{\Delta N} = g'_{\Delta \Delta} = 0.6$, we find $\chi \approx 0.8$ (Fig. 2). This is principally due to the large contribution of the ph Lindhard function which, at $\omega_\pi = 0$, is proportional to $p_F$ and therefore does not change appreciably if one reduces the density within a reasonable range. One obvious improvement is the use of an energy gap in the ph-spectrum at the Fermi surface. It accounts in an average way for the low-lying excitation spectrum of a finite nucleus [15]. Using a gap of 8.5 MeV, appropriate for the continuum threshold of the triton, leads to a reduction of $\chi$ in $N \neq Z$ nuclear matter but is still not able to describe the slope of $\Sigma_\pi$ inferred from the measurement (Fig. 2). This indicates that the use of the bulk matter Lindhard function is not appropriate for such a small nucleus and the kinematics probed in the experiment. Therefore, we do not attempt to calculate $\chi$ but rather use the above value $\chi = 0.31 \pm 0.22$ from experiment. This allows an extrapolation of the self energy to $\vec{q}_\pi = 0$ and to determine the mean density experienced by the virtual pion, with the result $\rho = \rho_p + \rho_n = (0.057 \pm 0.085)$ fm$^{-3}$ $\approx \frac{1}{3} \rho_0$, albeit with a large error. The self energy corresponding to the best fit is displayed in Fig. 2.

For further physical interpretation of the measurement we use guidance from ChPT to infer the effective $\pi^+$ mass at the density probed in the experiment. Given the above mentioned uncertainties in the use of the local density approximation for the medium modification of the pion in very light nuclei these results should be regarded as qualitative. The effective mass can be obtained from an extrapolation of the pion self energy to the mass shell. Up to second order in $\omega_\pi$ and $m_\pi$, the self energy of a charged pion in homogeneous, spin-saturated, but isospin-asymmetric nuclear matter in the vicinity of $\omega_\pi \approx m_\pi$ and for $\vec{q}_\pi = 0$ is given by the expansion

$$\Sigma_\pi(\omega_\pi, 0) = \left( -\frac{2(c_2 + c_3)\omega_\pi^2}{f_\pi^2} - \frac{\sigma_N}{f_\pi^2} \right) \rho + \frac{3}{4\pi^2} \left( \frac{3\pi^2}{2} \right)^{1/3} \frac{\omega_\pi^2}{4f_\pi^2} \rho^{4/3}$$

$$\pm \frac{\omega_\pi}{2f_\pi^2} (\rho_p - \rho_n) + \cdots,$$

where the $+/-$ signs refer to the respective charge state of the pion (see also [16]). The low-energy constants (LEC’s) $c_2$ and $c_3$ of the Chiral Lagrangian and the $\pi N$ sigma term $\sigma_N$ characterize the $\pi N$ interaction and are related to the $\pi N$ scattering lengths. We use $(c_2 + c_3) \times m_\pi^2 = -26$ MeV [14], but one should remark here that third-order corrections may change the LEC’s somewhat [17]. The pion self energy in Eq. (3) consists of two isoscalar parts proportional to $\rho$ and $\rho^{4/3}$, respectively, and an isovector part proportional to $(\rho_p - \rho_n)$. The latter is known as the "Tomozawa–Weinberg term" [18]. Based on PCAC arguments, it reflects the isovector dominance of the $\pi N$ interaction at $\omega_\pi = m_\pi$, where the isoscalar scattering length as given by the first coefficient in Eq. (3) vanishes at leading order. The second isoscalar term proportional to $\rho^{4/3}$ is caused by s-wave pion scattering from correlated nucleon pairs [19]. The sign of the Tomozawa–Weinberg term depends on the isospin...
asymmetry of the nuclear medium. In the present case of a virtual \( \pi^+ \) propagating in a triton-like medium with \( \rho_p - \rho_n = -\frac{1}{2} \rho \), the isovector term becomes attractive.

The effective \( \pi^+ \) mass \( m^\ast_{\pi^+} \) is deduced from the pole of the pion propagator at \( \tilde{q}_\pi = 0 \) which is determined by the solution of \( \omega^2_{\pi^+} - m^2_{\pi^+} = \Sigma(\omega_{\pi^+}, 0) = 0 \) with the self energy as given by Eq. (3). Using \( \rho = (0.057^{+0.085}_{-0.057}) \) fm\(^{-3} \), one obtains a mass shift \( \Delta m_{\pi^+} = m^\ast_{\pi^+} - m_{\pi^+} = (-1.7^{+1.7}_{-2.1}) \) MeV/\( c^2 \) when the \( \pi^+ \) propagates in \(^3\)H. It is interesting to compare the determined negative mass shift \( \Delta m_{\pi^+} \) with a positive mass shift \( \Delta m_{\pi^-} \) derived from deeply bound pionic states [20,21] in \(^{207}\)Pb and \(^{205}\)Pb with \( N/Z \approx 1.5 \). Itahashi et al. [21] have reported a strong repulsion of 23 to 27 MeV due to the local potential \( U_{\pi^+}(r) \) for a deeply bound \( \pi^- \) in the center of the neutron-rich \(^{207}\)Pb nucleus. Evaluating Eq. (3) for this case with \( \rho_p + \rho_n = \rho_0 \) and \( \rho_n/\rho_p = N/Z \approx 1.5 \) one calculates \( U_{\pi^+}(0) = \Sigma(\omega_{\pi^+}, 0)/\rho_p \approx 18 \) MeV. This is in good agreement with the findings of Ref. [16]. Yet, there remains the problem of a “missing repulsion” in the interpretation of the pionic atom data.

Fig. 3 shows the contributions to the pion mass shift in \(^3\)H: the two isoscalar contributions to \( \Sigma_\pi \) are both repulsive and increase the pion mass. One thus notices from Eq. (3) that at \( \omega_{\pi^+} = m^\ast_{\pi^+} \neq m_{\pi^+} \) already the isoscalar contribution to the self energy is sizeable. For a neutron-rich nucleus the isovector \( \pi N \) interactions are attractive (repulsive) for \( \pi^+(\pi^-) \) giving rise to a splitting of the mass shifts (contribution 3 in Fig. 3). In \(^3\)H, the isoscalar and isovector terms are compensating each other to a large extent, resulting in the very small decrease of the \( \pi^+ \) mass.

3.2. Modification of the \( \Delta \)

Most of the DWIA overestimate in the transverse channel (cf. Fig. 1) is removed by a medium modification of the \( \Delta \) isobar. The in-medium \( \Delta \) propagator is written [9] as \( [\sqrt{\omega - M_\Delta + i\Gamma_\Delta/2 - \Sigma_\Delta} - 1] \), where one introduces a complex self-energy term \( \Sigma_\Delta \) in the free \( \Delta \) propagator. Besides this explicit medium modification of the production amplitude also the DWIA formalism for the pion-nucleus rescattering effectively accounts for a \( \Delta \) modification in the medium [11]. The quantity \( \Sigma_\Delta \) has been deduced from an energy-dependent fit to a large set of \( \pi^0 \) photoproduction data [9,22] from \(^4\)He and also consistently describes recent photoproduction data from \(^{12}\)C, \(^{40}\)Ca and \(^{208}\)Pb [23]. The fitting procedure reported in [9] has been redone with the unitary phase excluded from the propagator in accordance with prescriptions often used in the \( \Delta \)-hole model [24]. The resulting \( \Delta \) self energy exhibits a dependence on the photon energy. Evaluated for the kinematics 1 and 2, which correspond to the photon equivalent energies \( E^\gamma = 392 \) and 376 MeV, respectively, the real and imaginary parts are Re \( \Sigma_\Delta \) \( \approx 50 \) and 39 MeV and Im \( \Sigma_\Delta \) \( \approx -36 \) and \(-29 \) MeV. Although quite large values are obtained in view of the small density \( \rho \approx \frac{1}{4} \rho_0 \), one should stress that the on-shell \( \Delta \) self energy at resonance position is numerically considerably smaller [6,25]. As a result, the agreement with the transverse cross section is significantly improved, although the experimental values are still overestimated by about 30%. The remaining discrepancy may be due to additional theoretical uncertainties. For example, the Fermi motion of the nucleons is effectively accounted for by a factorization ansatz [9]. An exact treatment might reduce the prediction of the transverse cross section by about 10%. A second uncertainty of the order of 10% concerns the knowledge of the elementary \( \pi^+ \) production amplitude at \( \theta_\pi = 0^\circ \). This kinematical region is not probed in photoproduction but may be accessible in the future with appropriate electroproduction data from the proton. Attributing the entire \( \Delta \) self energy to a mass shift \( \Delta M_\Delta \) and a width change \( \Delta \Gamma_\Delta \), we deduce an increase by 40 to 50 MeV and 60 to 70 MeV, respectively. These values seemly differ from our earlier results [5], where we have employed the parameterization from Ref. [26] which did not include the \( \Delta \)-hole interaction, giving Re \( \Sigma_\Delta \approx -14 \) MeV for
a mean $^3$He density of $\rho = 0.09$ fm$^{-3}$. On the other hand, the self-energy term of the present work is an effective parameter which incorporates the influence of the $\Delta$-spreading potential, Pauli- and binding effects as well as the $\Delta$-hole interaction including the Lorentz–Lorenz correction. This finally leads to the positive sign.

The effects of the medium modifications were also examined in the angular distribution of the produced pions in kinematics 2. The data along with the model calculations are shown in the l.h.s. of Fig. 4. The asymmetry of the combined distribution is due to a finite LT interference term. From the azimuthal dependence on $\phi_{\pi}$ for three polar angle bins $\theta_{\pi}$ we extract the LT interference term as a function of the pion emission angle $\theta_{\pi}$, as shown in the r.h.s. of Fig. 4 along with the comparison to the model calculations. It is obvious that only the full calculation, incorporating the medium modifications in the pion and $\Delta$ propagators, is able to reproduce the angular distributions.

4. Summary

In summary, in a kinematically complete experiment, we have measured the longitudinal and transverse cross section as well as the LT interference term for the first time in the $^3$He(e,e$'\pi^+$)$^3$H reaction. The high sensitivity of the electroproduction cross section shows clear evidence for self-energy corrections in both the pion and $\Delta$-isobar propagators and complements the large body of previous results from pion-nucleus data. Using ChPT we have extrapolated the pion self-energy determined from the present experiment to the mass shell to deduce the effective $\pi^+$ mass in $^3$H. Although qualitative, the results appear to be consistent with the theoretical analysis of deeply bound pionic atoms and the deduced effective $\pi^-$ mass [16]. In the transverse channel, the medium modification of the $\Delta$ isobar is also evident and the self-energy modifications inferred from the present measurements conform with $\pi^0$ photoproduction data over a wide mass range.

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